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THE DUODECIMAL BULLETIN

82;(98.)

Annual Award

See page 1#

Volume 41;(49.)
Number 1
11*8; (2000.)

THE DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City LI NY 11530-6793

=Annual Meeting Fri, Oct 6; 11*8(10/6/00) Hofstra University 3:30 PM.=

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, non profit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights & measures, & other branches of pure & applied science.

Membership dues are $12 (US) for one calendar year. Student Membership is $3 (US) per year, and a life Membership is $144 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530-6793.

THE
DUODECIMAL BULLETIN

Whole Number 82; (Eight Dozen Two)

Volume 41; (4 Dozen One)

Number 1;

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IN THIS ISSUE

VISIT OUR WEB SITE AT

http://members.xoom.com/harvestevia

Saturday, November 6, 11*7;(1999.)
Nassau Community College, Garden City, NY

Attendance: Prof. Alice Berridge, Prof. John Earnest, Chris Harvey, Dr. John Impagliazzo, Christina Scalise, Prof. Jay Schiffman, Prof. Gene Zirkel.

BOARD OF DIRECTORS MEETING

1. Gene Zirkel convened the meeting at 10:45 AM. The following Board members were present: Alice Berridge, John Earnest, John Impagliazzo, Christina Scalise, Jay Schiffman, and Gene Zirkel.

2. The minutes of the meeting of October 15;11*6;(10/17/98) were approved as published in *The Bulletin*.

3. The Nominating Committee (A. Berridge, J. Schiffman, and Pat Zirkel) presented the following slate of officers, which was elected unanimously:

Board Chair:	Gene Zirkel
President:	Jay Schiffman
Vice President:	John Earnest
Secretary:	Christina Scalise
Treasurer:	Alice Berridge

4. Appointments were made to the following Committees:

Annual Meeting Committee: Alice Berridge and Gene Zirkel
Awards Committee: Gene Zirkel, Patricia Zirkel, Jay Schiffman.
Volunteers to these committees are welcome at any time.

5. The following appointments were made:

Editor of *The Duodecimal Bulletin*: Jay Schiffman
Associate Editor: Gene Zirkel
Parliamentarian to the Board Chair: Christina Scalise

Other Business of the Board: The next Board Meeting will be held on Friday, October 6; 11*8;(10/6/00) at 3:30 PM at Hofstra University. After the business has been conducted, and the featured presentations there will be a complimentary buffet dinner. John Earnest agreed to present a PC/Web Page Demonstration/lecture designed to be appealing to a more generalized

audience. Chris Harvey agreed to follow the lecture with a demonstration of the new DSA Web Page which will be formally opened at that time

John Earnest led a discussion about marketing ideas for the Society. He suggested that a *Bulletin* and a letter be sent to heads of Mathematics departments at colleges and high schools in this area and maybe elsewhere. It was suggested that a donation be made to the L.I. Math Fair along with an advertisement for their program announcement. It was also suggested that NYSMATYC, AMATYC and MAA be approached to see if ads or announcements can be made about DSA.

FEATURED SPEAKER

Christina Scalise who has been working this past year to develop a web site for the Society introduced Chris Harvey, a student at Friends Academy, who has been working with Christina on the web site project. He discussed its status at this point and demonstrated at a terminal how it operates. There is a need to obtain a permanent server and there is hope that this can be done through Nassau Community College or Nassau Net. John Earnest has agreed to approach the Mathematics Department Chairperson at Nassau with this proposal. In the meantime members can access the site at: members.xoom.com/harvestevia. When the site is finalized all possible dozens keywords will be entered and officers will be listed as contact persons. Members are advised to contact Chris at e-mail address: til2csh@Hofstra.edu with suggestions or advice for improvement of the site.

It was thought that Andrews' son, Peter Andrews, might have material that would be suitable and useful for the site. At the moment *Excursions in Numbers* is the only article at the site.

The Board Meeting was adjourned at noon.

ANNUAL MEMBERSHIP MEETING

1. President Jay Schiffman gaveled the meeting to order at 12:10 PM.

2. The minutes of the meeting of October 15; 11*6;(10/17/98) were approved as published in *The Bulletin*.

3. Treasurer's Report - Alice Berridge

Alice presented Income Statements for the years 11*7, 11*6, 11*5 and 11*4

(1999-1996) for comparison, as well as Membership lists for the last two years and a listing of current members as reflected from the recent membership drive. Last year we gained one new Life Member and there is another new Life Member from the current drive. Six Life Members, and seven Regular Members made special contributions to the Society amounting to $41#;($599.) and $55;($65.), respectively. A second pitch for membership will be made soon. The Balance as of September 1; was $#0#;($1595.). Gene provided certificates, which Alice will send to the two newest Life Members.

4. Editor's Report - Jay Schiffman

Jay announced that he is working to get the next *Bulletin* finished for an early spring mailing. He is still anxious to receive articles from readers. Gene is now doing the copy for *The Bulletin* on disk thereby eliminating paste-up work, and thus cutting costs. This has been found to be much more efficient for the editors.

5. Annual Meeting Committee - Alice Berridge and Gene Zirkel

The next Annual Meeting will take place on Friday, October 6; 11*8(10/6/00) at Hofstra University at 3:30 PM.

6. Nominating Committee - Alice Berridge

The Committee presented the following slate for the class of 11**(2002): Ian B. Patten, Chris Harvey, Christina Scalise, and Dr. Tony Scordato. The slate was elected unanimously.

In addition, John Steigerwald was elected to the Class of 11*8 to replace John Hansen, Jr.

Alice Berridge, Jay Schiffman and Patricia Zirkel were proposed as the Nominating Committee for the coming year. They were elected unanimously.

Christina Scalise was appointed Parliamentarian to the Chair.

7. Awards Committee – Gene Zirkel

The DSA award for 11*7; will be presented to Nassau Community College in the form of a plaque indicating that: "For many years the Administration of Nassau Community College, the Faculty of the Mathematics Department and the Library Staff have helped the Society in its efforts to provide free literature

and assistance to students, teachers and other interested parties."

In addition, President Jay Schiffman was awarded the rank of Fellow of the Society in appreciation of all his efforts on behalf of DSA.

A stuffed panda, the six-fingered mascot of DSA, was presented to Christina Scalise for her efforts on behalf of the DSA Web Site.

7. Other Business:

Bob McPherson who had contacted the Board for corrections to his Certificate of Numeral Equivalence which he intends to file in the State of Florida needs to have a new set of signatures which will be sent out to him by Gene.

Gene informed members that there are 31 Life Members and that there are only seven remaining DSA members with two-digit numbers: member 11; Skip Scifres, member 14; Dallas H. Lien, member 32; Albert S. DeValve, member 4#; Robert McPherson, member 40; Paul Adams, member 67; Gene Zirkel, and member 37; Thomas O'Neill.

Gene reported that he has responded to about three dozen requests for literature this year and that generally there are many more requests. Members approved the new DSA stationery, which Gene has designed.

Ian Patten submitted Chapter 8 of a manuscript that he is working on for members to read and provide comment. Members agreed that they liked the tenor of the work and said that if he were to send other chapters we could consider them all for publication in *The Bulletin*. He would still be able to retain copyright privileges for future book publication.

<ins>Featured Speaker</ins>

DSA President Jay Schiffman, of Rowan University, Camden Campus, spoke on the topic "Dozenal and other cyclical patterns in the terminal digits of three recursive sequences in four different number bases."

President Schiffman

He considered the Fibonacci, the Lucas and the Tribonacci sequences in base dek, base twelve, octal and Hex. He used the TI 89, also the TI86, to demonstrate and graph the cyclical patterns. Members were very interested in the topic. One item in particular: the number 2^{67}-1 is not prime and Jay was able to produce the factors: 193707721. and 761838257287. He presented copies of "The Fibonacci numbers" from the site:

http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibtable.htmlhttp://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibtable.html

along with his *Bulletin* article (3#;1;11*6) "Three Recursive Sequences in Three Different Number Bases. Jay can be reached at

schiffman@camden.rowan.edu

for a copy of his report.

The meeting was adjourned at 2:30 PM.

Respectfully submitted,

Alice Berridge

F Emerson Andrews book, "New Numbers" had 3 editions:

☆First edition copyright 1935 by Harcourt, Brace & Company, Inc., NY

☆Second edition copyright 1944 by Andrews & published by Essential Books, NY

☆A third edition was published in England during WW II. The warehouse in which they were stored was destroyed by Nazi bombing.

Our archives contain copies of the first two editions. Do any of our readers possess a copy of the British edition? Please contact us if you do. If we can obtain the details such as the publisher and the date, we can request book search companies to look for it for us. Thanks.

THE ROLE OF CONGRUENCES IN PROVING HEXADECIMAL DIVISIBILITY TESTS

Jay L. Schiffman
Rowan University, Camden Campus

INTRODUCTION:

It is often useful to quickly determine when one integer is divisible by another without resorting to tedious long division. To cite an example in the decimal base, it is well known that an integer is divisible by 3 if the number formed by the sum of the digits (no matter how many digits the integer possesses) is divisible by 3, and conversely, if the number formed by the sum of the digits is divisible by 3, then the integer is divisible by 3.

The purpose of this article is to utilize congruences to furnish hexadecimal divisibility tests for an integer by each of the integers from One to Eighteen. Throughout the article, the set Z will denote the set of all integers. We initiate our discussion with the hexadecimal system of numeration. In addition to the ten symbols in the decimal system, we include the six alphabetical characters A, B, C, D, E, and F to correspond respectively to our decimal integers ten through fifteen. Hence the numeral $(A3C)_h$ indicates ten groupings of sixteen squared, three groupings of sixteen, and twelve units. One may express this via the mathematical equation $(A3C)_h = (10 \times 16^2) + (3 \times 16) + (12 \times 1) = (10 \times 256) + (3 \times 16) + (12 \times 1) = 2560 + 48 + 12 = 2620$ decimally. To convert the decimal numeral 2537 to hexadecimals, we initially divide 2537 by 16: 2537/16 = 158R.**9**. Next divide the new quotient 158 by 16: 158/16 = 9R.**14**. Now divide this new quotient 9 by 16: 9/16 = 0R.**9**, Finally divide the last quotient 0 by 9: 0/9 = 0. At this stage, we have completed our desired goal. Finally, read the sequence of remainders obtained in reverse order converting the decimal numeral 14 to the equivalent hexadecimal numeral E. Thus $2537 = (9E9)_h$.

Our next goal is to review the definition of congruence.

DEFINITION 1: For a, b, n $\in$ Z with $n \geq 2$, we write $a \equiv b \pmod{n}$ read "a is congruent to b, modulo n," to denote that a - b = kn. (The difference between a and b is a multiple of n) for some k $\in$ Z. This can be succinctly stated as follows: $a \equiv b \pmod{n}$ if $n \mid (a - b)$ where | denotes the division relation, and is read " n divides (a - b)". Otherwise we write $a \not\equiv b \pmod{n}$.

The number n is called modulus of the congruence. The term modulus is derived from the Latin meaning "little measure."

To illustrate our definition hexadecimally, 5 ≡ 21 (mod 7) (since 5 - 21 = -1C = -4 x 7) while 5 ≢ 16 (mod 3) (since 5 - 16 = -11 which is not an integral multiple of 3.)

An alternative definition of congruence is the following: a ≡ b (mod n) if and only if a and b have the same remainder when divided by n. Here a, b ∈ Z and n ∈ N where N connotes the set of positive integers. Hence 5 ≡ 21 (mod 7) since 5 and 21 both have remainder 5 upon division by 7 in the hexadecimal base.

The following theorem furnishes one with the properties of congruences which are essential for our work. The proof of Theorem 1 can be found in any standard number theory textbook.

THEOREM 1: For a, b, c, n ∈ Z with n ≥ 2:

(1). a ≡ a (mod n)

(2). If a ≡ b (mod n), then b ≡ a (mod n).

(3). If a ≡ b (mod n) and b ≡ c (mod n), then a ≡ c (mod n).

(4). If a ≡ b (mod n) and c ≡ d (mod n), then a + c ≡ b + d (mod n).

(5). If a ≡ b (mod n) and c ∈ Z, then ac ≡ bc (mod n).

(6). If a ≡ b (mod n) and c ≡ d (mod n), then ac ≡ bd (mod n).

(7). If a ≡ b (mod n) and k ∈ N, then $a^k \equiv b^k$ (mod n) where N = 1, 2, 3, . . ., the set of positive integers.

(8). Let a ≡ b (mod n). Then for all $c_0, c_1, \ldots, c_k \in Z$, the following congruence is valid (mod n):
$c_0 + c_1a + c_2a^2 + \ldots + c_ka^k \equiv c_0 + c_1b + c_2b^2 + \ldots + c_kb^k$

We present an illustration for each Part of Theorem 1 in the hexadecimal base.

(1). 5 ≡ 5 (mod 10) since 5 - 5 = 0 = 0 x 10.

(2). 10 ≡ 2 (mod 7) since 10 - 2 = E = 2 x 7. Now 2 ≡ 10 (mod 7) since 2 - 10 = -E = -2 x 7.

(3). 15 ≡ 6 (mod 3) since 15 - 6 = F = 5 x 3. Also 6 ≡ 3 (mod 3) since 6 - 3 = 3 = 1 x 3. Jointly these two congruences imply 15 ≡ 3 (mod 3) and 15 - 3 = 12 = 6 x 3.

(4). 23 ≡ 15 (mod E) since 23 - 15 = E = 1 x E. Also 57 ≡ 1F (mod E) since 57 - 1F = 38= 4 x E. Hence 23 + 57 ≡ 15 + 1F (mod E), or 7A ≡ 34 (mod E) since 7A - 34 = 5(E).

(5). Observe that 23 ≡ 11 (mod 6) since 23 - 11 = 12 = 3 x 6. Also 5 x 23 ≡ 5 x 11 (mod 6) or AF ≡ 55 (mod 6) since AF - 55 = 5A = F x 6.

(6). 38 ≡ 6 (mod 19) and 10 ≡ -9 (mod 19). Observe that 38≡ 6 (mod 19) since 38 - 6 = 32 = 2 x 19. In addition, 10 ≡ -9 (mod 19); for 10 - (-9) = 10 + 9 = 19 = 1 x 19. Now 38 x 10 ≡ 6(-9) (mod 19) since 380 ≡ -36 (mod 19) as 380 - (-36) = 380 + 36 = 3B6 = 26 x 19.

(7). 12 ≡ -3 (mod 7); for 12 - (-3) = 12 + 3 = 15 = 3 x 7. Now $12^3 \equiv (-3)^3$ (mod 7) since 16C8 ≡ -1B (mod 7) as 16C8 - (-1B) = 16C8 + 1B = 16E3 = 345 x 7.

(8). 10 ≡ 4 (mod 3) since 10 - 4 = C = 2 x 6. Now I claim C + 4 x 10 + A x $(10)^2$ ≡ C + 4 x 4 + A x $(4)^2$ (mod 3). This is equivalent to C + 40 + A x 100 ≡ C + 10 + A0 (mod 3). And A4C ≡ BC (mod 3) AS A4C - BC = 990 = 330 x 3.

Before presenting our next theorem, some machinery is required.

DEFINITION 2: The <u>greatest common divisor</u> of two integers is the largest integer which is a divisor of both. Two integers are said to be <u>relatively prime</u> or <u>co-prime</u> is their greatest common divisor is one. We denote the greatest common divisor of a and b by (a,b).

To illustrate, the greatest common divisor of the hexadecimal numerals F and 14 (decimally 15 and 20 respectively) is 5 since is the largest counting integer which is a divisor of both F and 14. On the other hand, the greatest common divisor of 8 and B (decimally 8 and 11 respectively) is 1 since 1 is the largest counting integer which is a divisor of both 8 and B. We state the next theorem without proof. The reader is invited to consult Reference (2) in the bibliography at the conclusion of this article for a formal proof of Theorem 2.

THEOREM 2: If d = (a,b), then there exist m, n ∈ Z such that d = ma + nb.

From Theorem 2, we obtain the following useful corollary:

Corollary 1: If (a,b) = 1, then there exist m,n ∈ Z such that ma + nb = 1.

Based on the Preceding Corollary, we derive Theorem 3 which provides the final touch needed for verifying the divisibility properties desired in the hexadecimal base. In the theorem, the notation a | b will denote that a divides b, where a, b ∈ Z.

THEOREM 3: Suppose (a,b) = 1, a | c and b | c. Then (a x b) | c. **PROOF:** Given (a,b) = 1, Corollary 1 guarantees us the existence of integers m and n satisfying the equation ma + nb = 1. Multiplying both sides of this equation by c yields mac + nbc.(1)Since a | c, we can find an integer s such that c = as. (2) Since b | c, we can find an integer t such that c = b*t. (3) Substituting (2) and (3) in (1) yields mabt + nabs = c. Now clearly ab | mabt and ab | nabs so that ab | (mabt + nabs) or ab | c as required.

It should be noted that since a, b, m, n, s, t ∈ Z so does mabt + nabs = c utilizing the closure Properties of Z under addition and multiplication.

We are now in position to achieve our main goal. We state the hexadecimal divisibility tests for the integers one to eighteen.

THEOREM 4: (The Hexadecimal Divisibility Tests for an integer n by each of the integers one to eighteen.)

(1). All integers n are divisible by 1.

(2). An integer n is divisible by 2 iff the last digit, a_0, is even; that is a_0 is either 0, 2, 4, 6, 8, A, C, or E.

(3). An integer n is divisible by 3 iff the digital sum $a_0 + a_1 + a_2 + \ldots + a_k$ is divisible by 3.

(4). An integer n is divisible by 4 iff the last digit, a_0, is either 0, 4, 8, or B.

(5). An integer n is divisible by 5 iff the digital sum $a_0 + a_1 + a_2 + \ldots + a_k$ is divisible by 5.

(6). An integer n is divisible by 6 iff n is divisible by both 2 and 3.

(7). An integer n is divisible by 7 iff $a_0 + 2a_1 - 3a_2 + a_3 + 2a_4 - 3a_5 + a_6 + \ldots$ is divisible by 7. (This sequence of multipliers repeats in the pattern 1, 2, -3, 1,... regardless of the number of digits.)

(8). An integer n is divisible by 8 iff the last digit, a_0, is either 0 or 8.

(9). An integer n is divisible by 9 iff $a_0 + 7a_1 + 4a_2 + a_0 + 7a_1 + 4a_2 + \ldots$ is divisible by 9. (This sequence of multipliers repeats in the pattern 1, 7, 4, 1, 7, 4, ... regardless of the number of digits.)

(10). An integer n is divisible by A iff n is divisible by both 2 and 5.

(11). An integer n is divisible by B iff $a_0 + 5a_1 + 3a_2 + 4a_3 - 2a_4 + a_5 + 5a_1 + 3a_2 + 4a_3 - 2a_4 + \ldots$ is divisible by B. (This sequence of multipliers repeats in the pattern 1, 5, 3, 4, -2, 1, 5, 3, 4, -2, ... regardless of the number of digits.)

(12). An integer n is divisible by C iff n is divisible by both 3 and 4.

(13). An integer n is divisible by D iff $a_0 + 3a_1 - 4a_2 + a_3 + 3a_1 - 4a_2 + \ldots$ is divisible by D. (This sequence of multipliers repeats in the pattern 1, 3, -4, 1, 3, -4, ... irrespective of the number of digits.)

[Continued on page 15;]

Win a Dozen Dollars!

In the article "Three Recursive Sequences in Three Different Number Bases", this *Bulletin*, Whole number 7*;(94.), Volume 3#(47.), Number 1, Page 18;(20.) There is an error in Table 1.

We will send a prize of one dozen dollars to the first person who finds *and* corrects it. Send your suggestions to Gene Zirkel, Six Brancatelli, West Islip LI NY 11795-2502.

LIQUID MEASURE

4 gills = 1 pint (pt)
2 pts = 1 quart (qt)
4 qts = 1 gallon (gal) = 231 cu"
31½ gals = 1 barrel (bbl)
2 bbls = 1 hogshead
1 cu' of water = 7.48 gals
1 cu' of water weighs ≈ 62½ lbs

LINEAR MEASURE

$^1/_{12}$" = 1 line
12" = 1'
3 feet = 1 yd
5½ yds = 16½' = 1 rd or pole
40 rds (660') = 1 furlong (fur)

320 rds (5280' = 8 furs) = 1 statute mile (m)

3 ms = 1 league
6' = 1 fathom
120 fathoms = 1 cable-length
$7^1/_3$ cables = 1 statute m
5280' = 1 statute m

6080.2' ≈ 1 geographical or nautical m
1 geographical m ≈ 1.15155 statute m

60 geographical m = 1° longitude at equator
360° = circumference of earth at equator

CIRCULAR MEASURE

60" = 1'
60" = 1°
90° = 1 quadrant
360° = 1 circumference
1° of earth's surface on a meridian ≈ 69 m

AVOIRDUPOIS WEIGHT

(Used in weighing all articles except drugs, gold, silver and precious stones)

$27^{11}/_{32}$ grains (gr) = 1 dram (dr)
16 drs = 437½ grs = 1 oz
16 ozs = 7000 grs = 1 lb
25 lbs = 1 quarter (qr)
4 qrs = 100 lbs = 1 hundredweight (cwt)
2000 lbs = 20 cwt = 1 ton (T)

NOTE-The ton and cwt given are those in common use in the US

2240 lbs = 1 long ton (L T)
13¼ cu' of air weighs 1 lb

NOTE-The grain has the same value in the Avoirdupois, Apothecaries and Troy systems

SQUARE MEASURE

144 square" = 1 square' (sq')
9 sq' = 1 sq yard (sq yd)

30¼ sq yds = 272¼ sq' = 1 sq rod (rd) = 1 perch
40 sq rds = 1 rood (R)
160 sq rds = 1 acre (A)
640 A = 1 sq mile (sq mi)
A square of area 1 A has a side of 208.71'
1 township = 36 sections each 1 mile^2
1 section = 640 As
¼ section = ½ mile^2 = 160 As
$^1/_8$ section = ½ mile x ¼ mile = 80 As
1 A = 4840 sq yds
1 A = a lot 208.71' sq

CUBIC MEASURE

1728 cu" = 1 cu'
27 cu' = 1 cu yard
16 cu' = 1 cord' of wood
128 cu'= 8 cord' = 1 cord of wood

NOTE-A cord of wood = a pile 8' x 4' x 4'

$24^3/_4$ cu' = 1 perch (P)

NOTE-A perch of stone or brick = a section 16½' x 1½' x 1'. The unit sometimes means 16½ cu' & sometimes ≈25 cu' (24.75)

40 cu' = 1 measurement ton US Shipping
42 cu' = 1 ton British Shipping
40' of round timber or 50' of hewn timber = 1 ton or load

APOTHECARIES' FLUID MEASURE

60 minims = 1 fluid dr
8 fluid drs = fluid oz
16 fluid ozs = 1 pt
8 pts = 1 gal

MISCELLANEOUS

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score
4 inches = 1 hand
Diameter x 3.1416 = circumference
Circumference x 0.3183 = diameter
Diameter squared x 0.7854 = area
Atmospheric pressure is 14.7 lbs per square" at sea level

Jean Kelly

Years ago many copy books contained a chart of 'Useful Information' such as found in our centerfold (pages 12; & 13;). You will notice that – with the exception of numbers such as 231 or 31½ – most of these practical units are divisible by 2, 3, 4 and that very few are divisible by ten.

The fractions have denominators of 2, 3, 2^2, 2^3, $2^2 3$ and 2^5. Nary a 5 or a ten in the lot.

Of the almost 5 dozen integers greater than 1, 4½ dozen are even numbers and most of these are divisible by 4 such as 40 rods = 1 furlong and 5280 feet = 1 mile. In addition almost 2 dozen have 3 as a factor such as 6 feet = 1 fathom and 5280 feet = 1 mile. At the same time less than 1½ dozen have ten as a factor but not twelve.

In addition, such charts also included a multiplication table for numbers from – not 1 to ten – but from 1 to a dozen. Since all students learned the multiplication algorithm for products of numbers with more than one digit, why did these charts continue on to twelve? (No, it was not because teachers were sadistic and wanted to punish students.) Maybe our ancestors were onto something as they created these arbitrary units of practical everyday measures? Just maybe!

JOTTINGS

We welcome 2 new members:

Chris Statler member number 366; from IOWA, and
Chris Harvey member number 367; our web page expert from NY.

In addition we welcome our newest Life member, Board member:

John Steigerwald 325 L; from Fanwood NJ

Continued from page 11;

(14). An integer n is divisible by E iff n is divisible by both 2 and 7.

(15). An integer n is divisible by F iff the digital sum is divisible $a_0 + a_1 + a_2 + \ldots + a_k$ is divisible by F. Also iff n is divisible by both 3 and 5.

(16). An integer n is divisible by 10 iff the last digit of n is 0.

(17). An integer n is divisible by 11 iff the alternating sum of the digits is divisible by 11. (That is n is divisible by 11 if $a_0 - a_1 + a_2 - a_3 + \ldots$ is divisible by 11.)

(18). An integer n is divisible by 12 iff n is divisible by both 2 and 9.

Before commencing with the proof of these divisiblity tests, it should be noted that a hexadecimal integer $a_k a_{k-1} \ldots a_1 a_0$ in expanded notation can be written as $n = a_k \times 10^k + a_{k-1} \times 10^{k-1} + \ldots + a_1 \times 10 + a_0$, where each $a_i \in Z$ such that $0 \leq a_i < 10$ and $k \in N$. In addition, $1 \equiv 1 \pmod{n}$ where $n \in N$ and $n \geq 2$.

PROOF OF THEOREM 4: (Note that since $a \equiv b \pmod{n}$ iff and b have the same remainder upon division by n, all of these proofs are reversible.)

(1). n is divisible by 1 since $n = 1 \times n$ for all $n \in Z$.

(2). To determine when n is divisible by 2, observe that $1 \equiv 1 \pmod{2}$ and $10 \equiv 0 \pmod{2}$. Hence $a_k \times 10^k \equiv 0 \pmod{2}$ for all $k \geq 1$. (Theorem 1 (vii)). so that $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + \ldots + a_k \times 10^k \equiv a_0 \pmod{2}$. That is, if n is divisible by 2, then the last digit, a_0, is even.

(3). To determine when n is divisible by 3, note that $1 \equiv 1 \pmod{3}$, $10 \equiv 1 \pmod{3}$, $10^2 \equiv 1 \pmod{3}$, and $10^k \equiv 1 \pmod{3}$ for each $k \in Z$, $k \geq 1$. Hence $a_k \times 10^k \equiv a_k \pmod{3}$ for each $k \geq 1$. Hence $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + \ldots + a_k \times 10^k \equiv a_0 + a_1 + a_2 + \ldots + a_k \pmod{3}$. *[Continued]*

Remember — your gift to the DSA is tax deductible

(8). To determine when n is divisible by 8, observe that $1 \equiv 1 \pmod 8$ and $10 \equiv 0 \pmod 8$ so that $10^k \equiv 0 \pmod 8$ for all $k \in Z$ with $k \geq 1$. Consequently, we have $a_k \times 10^k \equiv 0 \pmod 8$ for all $k \geq 1$. Hence $n = a_0 + a_1 \times 10 + a_2 \times 10;^2 + \ldots + a_k \times 10;^k \equiv a_0 \pmod 8$. We conclude that if n is divisible by 8, then the last digit, a_0, is either 0 or 8.

(4). To determine when n is divisible by 4, note that $1 \equiv 1 \pmod 4$ and $10 \equiv 0 \pmod 4$ so that $10^k \equiv 0 \pmod 4$ for each $k \in Z$, $k \geq 1$. Hence $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + \ldots + a_k \times 10^k \equiv a_0 \pmod 4$ which occurs if $a_0 = 0, 4, 8$, or C.

(5). To determine when n is divisible by 5, observe that $1 \equiv 1 \pmod 5$, $10 \equiv 1 \pmod 5$, $10^2 \equiv 1 \pmod 5$, and $10^k \equiv 1 \pmod 5$ for each $k \in Z$, $k \geq 1$. Hence $a_k \times 10^k \equiv a_k \pmod 5$ for each $k \in Z$, $k \geq 1$. Hence $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + \ldots + a_k \times 10^k \equiv a_0 + a_1 + a_2 + \ldots + a_k \pmod 5$.

(6). To determine when n is divisible by 6, note that if $2|n$ and $3|n$, then since $(2,3) = 1$, Theorem 3 guarantees $(2*3)|n$ or $6|n$.

(7). To determine when n is divisible by 7, note that $1 \equiv 1 \pmod 7$, $10 \equiv 2 \pmod 7$ so that $10^2 \equiv (2)^2 \equiv -3 \pmod 7$, $10^3 \equiv -6 \equiv 1 \pmod 7$, $10^4 \equiv 2 \pmod 7$, $10^5 \equiv -3 \pmod 7$, $10^6 \equiv 1 \pmod 7$ and the process of these multipliers repeats. In addition, $a_1 \times 10 \equiv 2a_1 \pmod 7$, $a_2 \times 10^2 \equiv -3a_2 \pmod 7$, $a_3 \times 10^3 \equiv a_3 \pmod 7$, $a_4 \times 10^4 \equiv 2a_4 \pmod 7$, $a_5 \times 10^5 \equiv -3a_5 \pmod 7$, $a_6 \times 10;^6 \equiv a_6 \pmod 7$ etc. so that $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + a_3 \times 10^3 + a_4 \times 10^4 + a_5 \times 10^5 + a_6 \times 10^6 + \ldots + a_k \times 10_k \equiv a_0 + 2a_1 - 3a_2 + a_3 + 2a_4 - 3a_5 + a_6 + \ldots \pmod 7$. (One should observe that since $10^2 \equiv -3 \pmod 7$ and $10^3 \equiv 1 \pmod 7$, then $10^5 = 10^2 \times 10^3 \equiv -3 \times 1 = -3 \pmod 7$.)

[Continued]

The dsa does NOT endorse any particular symbols for the digits ten and eleven. for uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and a semi-colon, or Humphrey point, as a unit point for base twelve.

Thus ½ = 0.5 = 0;6 and ¼ = 0.25 = 0;3 while ⅓ = 0.333... = 0;4

(9). To determine when n is divisible by 9, observe that $1 \equiv 1 \pmod 9$, $10 \equiv -2$ mod 9), $10^2 \equiv (2)^2 \equiv 4 \pmod 9$, $10^3 \equiv -2 \times 4 = -8 \equiv 1 \pmod 9$, $10^4 \equiv -2 \pmod 9$, $10^5 \equiv 4 \pmod 9$, $10^6 \equiv 1 \pmod 9$ and the sequence of multipliers repeats. In addition, $a_1 \times 10 \equiv -2a_1 \pmod 9$, $a_2 \times 10^2 \equiv 4a_2 \pmod 9$, $a_3 \times 10^3 \equiv a_3 \pmod 9$, $a_4 \times 10^4 \equiv -2a_4 \pmod 9$, $a_5 \times 10^5 \equiv 4a_5 \pmod 9$, $a_6 \times 10^6 \equiv a_6 \pmod 9$, etc. so that $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + a_3 \times 10^3 + a_4 \times 10^4 + a_5 \times 10^5 + a_6 \times 10^6 \equiv a_0 - 2a_1 + 4a_2 + a_3 - 2a_4 + 4a_5 + a_6 + \ldots \pmod 9$.

(A). To determine when n is divisible by A, note that if $2|n$ and $5|n$, then since $(2,5) = 1$, Theorem 3 assures us that $(2 \times 5)|n$ or $A|n$.

(B). To determine when n is divisible by B, note that $1 \equiv 1 \pmod B$, $10 \equiv 5 \pmod B$, $10^2 \equiv 3 \pmod B$, $10^3 \equiv 4 \pmod B$, $10^4 \equiv -2 \pmod B$, $10^5 \equiv 1 \pmod B$ and this sequence of multipliers continues in the same pattern. Consequently, $a_1 \times 10 \equiv 5a_1 \pmod B$, $a_2 \times 10^2 \equiv 3a_2 \pmod B$, $a_3 \times 10^3 \equiv 4a_3 \pmod B$, $a_4 \times 10^4 \equiv -2a_4 \pmod B$, $a_5 \times 10^5 \equiv a_5 \pmod B$, etc.

Hence $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + a_3 \times 10^3 + a_4 \times 10^4 + a_5 \times 10^5 \ldots + a_k \times 10^k \equiv a_0 + 5a_1 + 3a_2 + 4a_3 - 2a_4 + a_5 + \ldots \pmod B$.

[Continued]

The dsa does NOT endorse any particular symbols for the digits ten and eleven. for uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and a semi-colon, or Humphrey point, as a unit point for base twelve.

Thus ½ - 0.5 = 0;6

(C). To determine when n is divisible by C, note that if 3|n and 4|n, since (3,4) = 1, Theorem 3 guarantees that (3 x 4)|n or C|n.

(D). To determine when n is divisible by D, note that $1 \equiv 1$ (mod D), $10 \equiv 3$ (mod D), $10^2 \equiv 3 \times 3 = 9 \equiv -4$ (mod D), $10^3 \equiv 3 \times (-4) = -C \equiv 1$ (mod D), etc. It immediately follows that $a_1 \times 10 \equiv 3a_1$ (mod D), $a_2 \times 10^2 \equiv -4a_2$ (mod D), $a_3 \times 10^3 \equiv a_3$ (mod D), and so forth. Hence $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + a_3 \times 10^3 + \ldots + a_k \times 10^k \equiv a_0 + 3a_1 - 4a_2 + a_3 + \ldots$ (mod D).

(E). To determine when n is divisible by E, note that if 2|n and 7|n, then since (2,7) = 1, Theorem 3 yields (2 x 7) |n or E|n.

(F15). To determine when n is divisible by F, note that if 3|n and 5|n, then since (3,5) = 1, invoke Theorem 3 to obtain (3 x 5)|n or F|n. (One could alternatively construct an argument to show that n is divisible by F if the number consisting of the digital sum likewise enjoys this property. (This is immediate since $10^k \equiv 1$ (mod F) for all $k \in N$.)

(10). To determine when n is divisible by 10, note that $1 \equiv 1$ (mod 10), $10 \equiv 10 \equiv 0$ (mod 10), and $10^k \equiv 0$ (mod 10) for each $k \in N$, so that $a_k \times 10^k \equiv 0$ (mod 10) for each $k \in N$. Consequently $n = a_0 + a_1 \times 10 + a_2 \times 10^2 + \ldots + a_k \times 10^k \equiv a_0$ (mod n). Conclude that n is divisible by 10 if the last digit $a_0 = 0$

(11). To determine when n is divisible by 11, note $1 \equiv 1$ (mod 11), $10 \equiv 10 \equiv -1$ (mod 11), $10^3 \equiv (-1) \times (-1) = 1$ (mod 11), $10^3 \equiv 1 \times (-1) = -1$ (mod 11), $10^4 \equiv (-1) \times (-1) = 1$ (mod 11). Thus $a_k \times 10^k \equiv a_k$ if k is even and $a_k \times 10^k \equiv -a_k$ if k is odd. Hence $a_0 + a_1 \times 10 + a_2 \times 10^2 + a_3 \times 10^3 + a_4 \times 10^4 + \ldots \equiv a_0 - a_1 + a_2 - a_3 + a_4 - \ldots$ (mod 11).

(12). To determine when n is divisible by 12, observe that if 2|n and 9|n, the relatively prime character of 2 and 9 guarantees us that (2 x 9)|n or 12|n.

We conclude by illustrating Theorem 4.

EXAMPLE: The hexadecimal integer BAF450 is divisible by each of the integers 1-12. Let us consider various categories.

While it is immediate that BAF450 is divisible by 1, BAF450 is divisible by the integers 2, 4, 8, and 10 since the last digit is 0.

To show that BAF450 is divisible by 3 and 5, note that the digital sum $a_0 + a_1 + a_2 + a_3 + a_4 + a_5 = 0 + 5 + 4 + F + A + B = 2D$. Now $2D \equiv 0$ (mod 3) and $2D \equiv 0$ (mod 5). (Observe that $2D \equiv 0$ (mod 3), since 3 | (2D - 0) or 3 | 2D; for 2D = 3 x F. (Similarly one notes that $2D \equiv 0$ (mod 5), since 5 | (2D - 0) or 5 | 2D as 2D = 5 x 9). To show that BAF450 is divisible by 11, note that $a_0 - a_1 + a_2 - a_3 + a_4 - a_5 = 0 - 5 + 4 - F + A - B = -11 \equiv 0$ (mod 11) since 11 | (-11 - 0) or 11 | -11; for -11 = 11 x (-1). BAF450 is divisible by 7 since 0 + 2 x 5 - 3 x 4 + F + 2 x A - 3 x B = 0 + A - C + F + 14 - 21 = 0 ≡ 0 (mod 7) since 7 | (0 - 0) or 7 | 0; for 0 = 7 x 0. BAF450 is divisible by 9 since 0 -2 x 5 + 4 x 4 + F -2 x A + 4 x B = 0 - A + 10 + F - 14 + 2C = 2D ≡ 0 (mod 9). To see that this congruence is indeed valid, 2D ≡ 0 (mod 9) since 9 | (2D - 0) or 9|2D since 2D = 9 x 5. BAF450 is divisible by D since $a_0 + 3a_1 - 4a_2 + a_3 + 3a_4 - 4a_5 = 0 + 3 \times 5 - 4 \times 4 + F + 3 \times A - 4 \times B = 0 + F - 10 + F + 1E - 2C = 0 \equiv 0$ (mod D) since D | (0 - 0) or D | 0 as 0 = D x 0. BAF450 is divisible by 11 as $a_0 - a_1 + a_2 - a_3 + a_4 - a_5 = 0 - 5 + 4 - F + A - B = -11 \equiv 0$ (mod 11) since 11 | (0 - 11) or 11 | -11 by virtue of the equation -11 = 11 x (-1).

Finally BAF450 is divisible by 6 since BAF450 is divisible by both 2 and 3, BAF450 is divisible by A since it is divisible by both 2 and 5, BAF450 is divisible by C since it is divisible by both 3 and 4, BAF450 is divisible by both 2 and 7 and thus divisible by E, BAF450 is divisible by both 3 and 5 and hence divisible by F, and to cap our solution, BAF450 is divisible by 2 and 9 and consequently divisible by 12. *[Continued]*

THE FOLLOWING ARE NOW AVAILABLE FROM THE SOCIETY

It is indeed fascinating to note that BAF450 = $2^4 \times 3^2 \times 5 \times 7 \times B \times D \times 11$ and is the minimal hexadecimal integer to enjoy divisibility by each of the integers from 1 to 12. Decimally, BAF450 = 12,252,240.

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THE ANNUAL AWARD FOR 11*7;

This year's *Ralph Beard Annual Award* - named in honor of one of our founders - was presented to Nassau Community College in recognition of their long time support of our efforts to help teachers and students thruout the world.

The year 11*7; marks the first time that the award was given to an institution rather than to an individual.

It was presented to the college by Gene Zirkel, Alice Berridge and John Earnest at a meeting of the Department of Mathematics and Computer Science on January 23; 11*8 (1/27/2000).

The text of the plaque appears on the following page.

Confusion

What would you think of an addition which read 11 + 2 = 31?

Ancient tablets from the area surrounding the city of Babylon are written in cuneiform. (These writings are often mislabeled "Babylonian".) The numerals depicted are written in base 5 dozen, and to add to the problem of deciphering them, they use neither a zero nor a fraction point.

Thus the numeral 12F, where F is the usual hexadecimal symbol used for the digit 13; (15.) might represent numerals such as 12F, 102F, 12.F, 12.0F, ... Translating these into our more usual dozenal (and decimal) notation we have:

12F	$= 1(50^2) + 2(50) + 13$	= 21#3	= (3735.)
102F	$= 1(50^3) + 0(50^2) + 2(50) + 13$	= *50#3	= (216,135.)
12.F	$= 1(50) + 2 + 13/50$	= 52;3	= (62.25)
12.0F	$= 1(50) + 2 + 0/50 + 13/50^2$	= 52;00 7249 7249 ...	= (62.004166666...)

And so our original question about addition if found on a cuneiform tablet might represent any of the following:

11	1001	1.01
20	2000	2.00
31	3001	3.01

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Given With Gratitude By The Board Of Directors.
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11*7; 1999.

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December 8,1999

Professor Gene Zirkel
The Dozenal Society of America
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Dear Gene,

On behalf of Nassau Community College, I want to thank you for the honor of the annual award being presented to the College from the Dozenal Society of America. I'm glad that the College has been supportive to the Society in its many endeavors.

Please convey my appreciation to all the members of the Dozenal Society for this honor.

Sincerely,

Sean A. Fanelli

President

SAF:la

WHY CHANGE?

WHY CHANGE? This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, people's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance. Then, in a related development, people awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, the most advantageous base should be used, the one best suited to the task at hand. (Similar to computer scientists use of binary, hexadecimal or octal - whichever is most convenient.) If this were done, duodecimals would progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited. □

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