Primel Metrology, Part II

~ *by John Volan* ~

This article was originally published in Duodecimal Bulletin, Whole Number ↊4$_z$.[1] *However, this version substantially revises the original, making changes in formatting and style of presentation, and including some expanded content. The most significant change is in the operator diacritics used in formulas to distinguish symbols representing "true·angles" from conventional symbols representing "dimensionless" angles.*[2]

In the first article in this series,[3] I introduced the Primel metrology (brand prefix **prime·**, brand mark ⊡), a coherent, dozenal-metric, day/gravity/water-based system of measurement which I have been developing for a number of years. I covered the basic units of mechanics and thermodynamics. I also introduced the concepts of "quantitel" unit names, metrology brand prefixes and brand marks, scaling prefixes using Systematic Dozenal Nomenclature[4] (SDN), and "colloquial" or "organic" unit names. This article begins covering more advanced topics, including reciprocal units and angular mechanics. Future articles will cover yet more advanced topics.

"Quantitelic" Units Inspired by ISO-31

ISO-31[5] is a standard published first published in 11↊0$_z$ (1992$_d$) by the International Organization for Standardization (ISO).[6] (It was superceded in 11↊9$_z$ (2001$_d$) by ISO/IEC 80000[7].) Among many other things, it coined several new English words for reciprocals of certain quantities. This regularized some terminology that had previously been more ad-hoc:

ISO-31 Coinage	Existing Terminology	Technical Meaning
massic *quantity*	**specific** *quantity*	*quantity* divided by associated mass
volumic *quantity*	[**volumic**] *quantity* **density**	*quantity* divided by associated volume
areic *quantity*	**surface** *quantity* **density**	*quantity* divided by associated area
lineic *quantity*	**linear** *quantity* **density**	*quantity* divided by associated length

This scheme takes a quantity term (such as *mass*) and applies a common *-ic* suffix to turn it into its reciprocal (*massic*), which then can act as a modifier on some other *quantity*. For instance, my previous article replaced the term *specific thermal capacity* with *massic heatability*, which measures *heatability* per *mass* of a given substance.

Primel extends this notion to the unit system, turning any unit name into its reciprocal by appending the **-ic** suffix. The **-ic** suffix may be abbreviated with a backslash (\) to indicate that the "quantitelic" unit places the original "quantitel" as a denominator "under" whatever quantity follows it (if any). For instance:

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1 https://dozenal.org/duodecimal-bulletin-0A4

2 This article annotates decimal numerals with a "d" subscript, dozenal numerals a "z" subscript. See https://dozenal.org/article-volan-base-annotation-schemes.

3 https://dozenal.org/article-volan-primel-metrology

4 https://dozenal.org/article-volan-systematic-dozenal-nomenclature-summary

5 https://en.wikipedia.org/wiki/ISO_31

6 https://en.wikipedia.org/wiki/International_Organization_for_Standardization

7 https://en.wikipedia.org/wiki/ISO/IEC_80000

Quantity	Primel Quantitel	Quantitelic Reciprocal	Recip Abbrev	Metric Equivalent
length	**prime·lengthel**	**prime·lengthelic**	⊡lgℓ\	$\approx 1.2192024384_{\mathrm{d}}$ per cm
area	**prime·areanel**	**prime·areanelic**	⊡arℓ\	$\approx 1.4864545858_{\mathrm{d}}$ per cm^2
volume	**prime·volumel**	**prime·volumelic**	⊡vmℓ\	$\approx 1.8122890556_{\mathrm{d}}$ per cm^3
mass	**prime·massel**	**prime·masselic**	⊡msℓ\	$\approx 1.8123398011_{\mathrm{d}}$ per gram

The previous article identified the **prime·masselic·heatabilitel** (abbreviated ⊡msℓ\htbℓ) as Primel's coherent unit of massic heatability. We will see more examples of quantitelic units as we proceed.

Primel Units of Angular Mechanics

Angular mechanics (also known as *rotational mechanics*) is the branch of classical mechanics dealing with objects rotating around a fixed axis.

Angles. Angular mechanics introduces *plane angle* (symbolized θ), as a distinct type of physical quantity to be measured. For thousands of years, people have been measuring angles using exclusive tools, such as protractors, compasses, sextants, theodolites, and the like. We express angle measurements using distinct units not applicable to any other type of quantity. These include the **full angle** or **turn** (abbreviated tr or ⊙) and various rational subdivisions of the turn, such as the **straightangle** ($\frac{\odot}{2}$), **quadrant** ($\frac{\odot}{4}$), **sextant** ($\frac{\odot}{6}$), **octant** ($\frac{\odot}{8}$), as well as the customary **degree** (°), **minute** (′), **second** (″). However, Primel prefers using power prefixes from SDN to define subunits of the turn, including the **uncia·turn** (u↓⊙), **bicia·turn** (b↓⊙), **trici·turn** (t↓⊙), **quadcia·turn** (q↓⊙), **pentcia·turn** (p↓⊙), **hexcia·turn** (h↓⊙), etc.

The Radian. As mentioned in the last issue, for purposes of physics, the **radian** (abbreviated rad) is the most appropriate choice for a *coherent* unit of plane angle. This is defined as an angle which subtends a circular arc of length equal to the radius of the enclosing circle.

Although the turn is obviously an important angular unit, and has intuitive appeal, it is not the coherent unit of angle. It is equivalent to τ radians:

$$\mathrm{tr} = \odot = \tau \ \mathrm{rad} \qquad \text{where}: \qquad \tau = 2\pi \approx 6.3494169678635_{\mathrm{z}}$$

Rearranging, we get this value for the radian in turns (as well as customary degrees):

$$\mathrm{rad} \approx 0.1↊↋02↊144357↊_{\mathrm{z}} \odot \approx 57.2957795130823_{\mathrm{d}}{}^{\circ}$$

The Squaradian or Steradian. In some cases, we will need to deal with angles squared. Primel coins **squaradian** (abbreviated sr) as the coherent unit for square angle, equivalent to the square of the radian. A synonym for this unit, useful in the context of spherical geometry, is the **steradian** (also abbreviated sr). The steradian is the coherent unit of *solid angle* (symbolized Ω).

$$\mathrm{sr} = \mathrm{rad}^2$$

A **spat** (abbreviated sp or ⊕) is a "full" solid angle covering the entire spherical space surrounding a given vertex. This is σ steradians:

$$\mathrm{sp} = \oplus = \sigma \ \mathrm{sr} \qquad \text{where}: \qquad \sigma = 2\tau = 4\pi \approx 10.696931713↋06↊_{\mathrm{z}}$$

Rearranging, we get this value for the the steradian in spats:

$$\mathrm{sr} \approx 0.0↋5615082189↊↊_{\mathrm{z}} \oplus$$

Angular Dimensionality. Primel asserts that plane angle is a distinct and sensible physical phenomenon, with an irreducible dimensionality of its own, not commensurate with any other type of physical quantity. Consequently, the radian, as the coherent unit of plane angle, constitutes another of Primel's "mundane realities."

Surprisingly, the notion that angles are dimensioned quantities is a controversial position, because the International System of Units[8] (SI), as well as many mainstream mathematicians, consider angles to be *dimensionless* quantities. Absurdly, the radian is actually equated with a pure number, i.e. rad = 1.

This stems from the idea that the measure θ of an angle is equivalent to the ratio of the length s of its subtended arc to the length r of its radius of rotation:

$$\theta = s/r \qquad s = r\theta \qquad r = s/\theta$$

Length over length yields a dimensionless quantity. But this treatment of angles leads to difficulties and inconsistencies across all the quantities of angular mechanics. Radians inexplicably appear and disappear from equations in ad-hoc ways not explicitly driven by the strict algebraic laws of dimensional analysis.

This has been so troubling that at least a dozen scientists since 1154_z (1936_d)[9] have published papers advocating for angle to become a first-class dimension. They have offered various schemes to reconcile the inconsistencies in order to render the equations of angular mechanics "dimensionally homogeneous."

Nevertheless, the status quo among mainstream mathematicians as well as scientists using SI is that angles are dimensionless quantities. All symbols such as θ representing angles are considered dimensionless. Moreover *all* the angle units mentioned above are dimensionless numbers as well, with the radian in particular indistinguishable from the dimensionless number 1. The turn is indistinguishable from the dimensionless circle constant τ, an irrational number. Other units represent fractions of this:

Unit	Abbrev	Dimensionless Value
turn	$\odot$	$= \tau \approx 6.349416967↋635_z$
straightangle	$\frac{\odot}{2}$	$= \frac{\tau}{2} \approx 3.18480943↋919_z$
quadrant	$\frac{\odot}{4}$	$= \frac{\tau}{4} \approx 1.6↊24048 47↋↊6↊_z$
sextant	$\frac{\odot}{6}$	$= \frac{\tau}{6} \approx 1.0696831713↋07_z$
octant	$\frac{\odot}{8}$	$= \frac{\tau}{8} \approx 0.951202423↋↋35_z$
degree	$^\circ$	$= \frac{\tau}{260_z} \approx 0.02617↊↋30397↊4739_z$
minute	$'$	$= \frac{\tau}{10{,}600_z} \approx 0.000604707 31↋81568_z$
second	$''$	$= \frac{\tau}{526{,}000_z} \approx 0.00001258 73↊2965114_z$
uncia·turn	u↓$\odot$	$= \frac{\tau}{10_z} \approx 0.6349416967↋635_z$
bicia·turn	b↓$\odot$	$= \frac{\tau}{100_z} \approx 0.06349416967↋635_z$
tricia·turn	t↓$\odot$	$= \frac{\tau}{1{,}000_z} \approx 0.006349416967↋635_z$
quadcia·turn	q↓$\odot$	$= \frac{\tau}{10{,}000_z} \approx 0.0006349416967↋635_z$
pentcia·turn	p↓$\odot$	$= \frac{\tau}{100{,}000_z} \approx 0.00006349416967↋635_z$
hexcia·turn	h↓$\odot$	$= \frac{\tau}{1{,}000{,}000_z} \approx 0.000006349416967↋635_z$

It's clear we need a different approach to the dimensionality of angles.

True·Angles. The approach I propose is to leave the status quo as it is, and instead distinguish a new concept: **true·angle**, i.e. a first-class dimension for angles, distinct from any other type of quantity. The **true·radian** (abbreviated ⓣrad) is the coherent unit of true·angle. It is identical in magnitude to the radian, but bearing true·angular dimension. Similarly, the **true·turn** (abbreviated

[8] See https://en.wikipedia.org/wiki/International_System_of_Units.

[9] See https://en.wikipedia.org/wiki/Radian#Dimensional_analysis for discussion and references.

ⓣtr or ⓣ⊙) is identical to the turn, but with true·angular dimension. Its dozenal divisions using SDN scaling prefixes are the **true·uncia·turn** (ⓣu↓⊙), **true·bicia·turn** (ⓣb↓⊙), **true·tricia·turn** (ⓣt↓⊙), and so forth. Other angle units may similarly be given true·angle versions.

Note that this **true·** prefix, and its abbreviation ⓣ, resemble my notion of a brand prefix for a metrology. Why am I introducing this, rather than simply using Primel's **prime·** prefix, and its abbreviation ⊡? The reason is that true·angles and true·radians are not exclusive to the Primel metrology. Any metrology can apply this treatment of angular measure. It can be argued that the radian is necessarily the coherent unit of angle, regardless of metrology. Consequently the "branding" for true·angular measures ought to be universal rather than metrology-specific.

However, just like with branding for a metrology, the "true" branding for angles can be made optional: In a context where all angles are assumed to bear first-class dimension, we can declare that as a global default, and omit the branding. In fact, the description of angular units in the previous article can be understood as written under the implicit assumption of such a global default. That said, this article will refrain from doing this, precisely in order to address the subtle distinctions between the SI and Primel approaches to angular measures and angular mechanics.

True·Radian Operator and True·Radianic Operator. In physical formulas, it will be necessary to distinguish symbols representing true·angles from symbols representing dimensionless angles. To accomplish this, I will use a variant of an approach proposed by Jacques Romain in 1176_z (1962_d).[↊]

Let an overset "frown" diacritic ($\frown$) be defined as a **true·radian operator**, which may be applied to any quantity q, to multiply it by one true·radian:

$$\overset{\frown}{q} = q \times ⓣ\text{rad}$$

Applying ISO-31 reciprocal terminology, let an underset "smile" diacritic ($\underset{\smile}{\ }$) be defined as a **true·radianic operator**, that can divide any quantity q by one true·radian:

$$\underset{\smile}{q} = \frac{q}{ⓣ\text{rad}} = q \times ⓣ\text{rad}^{-1}$$

As a trivial case, if we let $q = 1$ (the pure number one), then:

$$\overset{\frown}{1} = ⓣ\text{rad} \qquad \underset{\smile}{1} = ⓣ\text{rad}^{-1}$$

The reciprocal unit ⓣrad^{-1} itself can be called a **true·radianic** (abbreviated ⓣrad\).

Note that both of these operator symbols are curved arcs, suggestive of the fact that we are dealing with rotations. They depict only partial rotations, suggestive of a radian arc, rather than a full turn. Placing the "frown" over a symbol and the "smile" under a symbol is suggestive that the former places a true·radian in the numerator while the latter places a true·radian in the denominator. The fact that these symbols are vertical mirror images suggests that they represent reciprocals of each other.

Based on these definitions, the true·radian and true·radianic operators follow fundamental laws of algebra. Per the associative law, applying either operator to a product of factors is equivalent to applying it just one of the factors:

$$\overset{\frown}{\overbrace{p \times q}} = \overset{\frown}{p} \times q = p \times \overset{\frown}{q} \qquad \underset{\smile}{\underbrace{p \times q}} = \underset{\smile}{p} \times q = p \times \underset{\smile}{q}$$

Per the distributive law, applying either of these operators to a sum of terms is equivalent to applying it to each of the terms individually:

$$\overset{\frown}{\overbrace{p + q}} = \overset{\frown}{p} + \overset{\frown}{q} \qquad \underset{\smile}{\underbrace{p + q}} = \underset{\smile}{p} + \underset{\smile}{q}$$

[↊] Romain, Jacques E. (July 1962_d). "Angle as a fourth fundamental quantity". *Journal of Research of the National Bureau of Standards Section B*. **66B** (3): 97. Freely accessible at `https://nvlpubs.nist.gov/nistpubs/jres/66B/jresv66Bn3p97_A1b.pdf`. He proposed a bracket notation $\langle q \rangle$ rather than a diacritic, as essentially a true·radianic operator. (However, he did not also define a true·radian operator.)

Given a conventional dimensionless angle θ, now we can use $\overset{\frown}{\theta}$ to represent the equivalent true·angle, with first-class physical dimension. Like any other dimensioned quantity, $\overset{\frown}{\theta}$ retains its intrinsic value no matter what true·angular units it is measured with. On the other hand, θ is now the dimensionless measure quantity when $\overset{\frown}{\theta}$ is specifically measured in true·radians:

$$\overset{\frown}{\theta} = \theta \text{ ⓣrad}$$

Rearranging this, we can say:

$$\frac{\overset{\frown}{\theta}}{\text{ⓣrad}} = \underset{\smile}{\overset{\frown}{\theta}} = \theta$$

In other words, when the true·radian and true·radianic operators are both applied, they cancel each other out.

Let's now revisit the equations that SI (and mainstream mathematicians) use to define an angle in terms of radius and arc length:

$$\theta = s/r \qquad s = r\theta \qquad r = s/\theta \qquad \text{(SI)}$$

As previously described, θ here represents a dimensionless quantity. In contrast, Primel endeavors to work only with true·angles, so we must apply the true·radian operator to θ. But if we want to keep these equations dimensionally balanced, we must apply the true·radianic operator to r:

$$\overset{\frown}{\theta} = s/\underset{\smile}{r} \qquad s = \underset{\smile}{r}\overset{\frown}{\theta} \qquad \underset{\smile}{r} = s/\overset{\frown}{\theta} \qquad \text{(Primel)}$$

What is the significance of $\underset{\smile}{r}$? It no longer means simply the *radius* of rotation, i.e., the length from the axis of rotation to any point on the circle. Now it means something subtler.

Radiality and the Radiel. I have come to call this new quantity the *radiality* of the rotation (symbolized $\underset{\smile}{r}$). It is the ratio of a length, specifically the radius, to a true·angle, specifically one true·radian (ⓣrad). However, as shown in the equation $\underset{\smile}{r} = s/\overset{\frown}{\theta}$, we can also describe $\underset{\smile}{r}$ as the ratio of any arc length s around the circle, to the true·angle $\overset{\frown}{\theta}$ subtended by that arc.

Primel's coherent unit of radiality is the **prime·radiel**[ε] (abbreviated ⊡rdℓ), defined as one prime·lengthel per true·radian:

$$\text{⊡rd}\ell = \frac{\text{⊡lg}\ell}{\text{ⓣrad}} = 8.20208\overline{3}_{\text{z}} \, \frac{\text{mm}}{\text{ⓣrad}}$$

Scalings of this unit, along with their colloquial synonyms, include:

Unit	Abbrev	Colloquial	Abbrev	Derivation	SI Equivalent
prime·radiel	⊡rdℓ	**prime·morsel·radiality**	⊡mo·rd	⊡mo·lg/ⓣrad	8.202083$_{\text{d}}$ mm/ⓣrad
prime·unqua·radiel	⊡u↑rdℓ	**prime·hand·radiality**	⊡hd·rd	⊡hd·lg/ⓣrad	98.425$_{\text{d}}$ mm/ⓣrad
prime·biqua·radiel	⊡b↑rdℓ	**prime·ell·radiality**	⊡ℓ·rd	⊡ℓ·lg/ⓣrad	1.1811$_{\text{d}}$ m/ⓣrad
prime·triqua·radiel	⊡t↑rdℓ	**prime·habital·radiality**	⊡hb·rd	⊡hb·lg/ⓣrad	14.1732$_{\text{d}}$ m/ⓣrad
prime·quadqua·radiel	⊡q↑rdℓ	**prime·stadial·radiality**	⊡ς·rd	⊡ς·lg/ⓣrad	170.0784$_{\text{d}}$ m/ⓣrad
prime·pentqua·radiel	⊡p↑rdℓ	**prime·dromal·radiality**	⊡dr·rd	⊡dr·lg/ⓣrad	2.0409408$_{\text{d}}$ km/ⓣrad
prime·hexqua·radiel	⊡h↑rdℓ	**prime·itineral·radiality**	⊡itn·rd	⊡itn·lg/ⓣrad	24.4912896$_{\text{d}}$ km/ⓣrad

Here, the choice of modifier in each colloquial name corresponds to the Primel length unit appearing in the numerator in the Derivation column.

[ε] Contraction of **prime·radialitel**.

Curvature and the Curvel. The reciprocal of *radiality* is a quantity called *curvature*. This is usually symbolized κ, but given the conventional treatment of angle as dimensionless, the dimensionality of κ is generally indistinguishable from reciprocal length. To correct this in terms of true·angles, we must apply the true·radian operator to yield true·curvature $\overset{\frown}{\kappa}$:

$$\overset{\frown}{\kappa} = \frac{1}{\underset{\smile}{r}}$$

The relationship of true·curvature $\overset{\frown}{\kappa}$ to true·angle $\overset{\frown}{\theta}$ and arc length s is thus characterized by these equations:

$$\overset{\frown}{\theta} = \overset{\frown}{\kappa} s \qquad s = \frac{\overset{\frown}{\theta}}{\overset{\frown}{\kappa}} \qquad \overset{\frown}{\kappa} = \frac{\overset{\frown}{\theta}}{s}$$

The coherent unit of true·curvature in Primel is the **prime·curvel**[10] (abbreviated ⊡cvℓ). This is equal to one true·radian per prime·lengthel:

$$⊡\text{cv}\ell = \frac{ⓣ\text{rad}}{⊡\text{lg}\ell} \approx 121.9202438404877_{\text{d}} \frac{ⓣ\text{rad}}{\text{m}}$$

Scalings of this unit, along with their colloquial synonyms, include:

Unit	Abbrev	Colloquial	Abbrev	Derivation	SI Equivalent
prime·curvel	⊡cvℓ	**prime·morsel·curvature**	⊡mo·cv	ⓣrad/⊡mo·lg	≈ $121.9202438404877_{\text{d}}$ ⓣrad/m
prime·uncia·curvel	⊡u↓cvℓ	**prime·hand·curvature**	⊡hd·cv	ⓣrad/⊡hd·lg	≈ $10.160020320041_{\text{d}}$ ⓣrad/m
prime·bicia·curvel	⊡b↓cvℓ	**prime·ell·curvature**	⊡ℓ·cv	ⓣrad/⊡ℓ·lg	≈ $846.668360003387_{\text{d}}$ ⓣrad/km
prime·tricia·curvel	⊡t↓cvℓ	**prime·habital·curvature**	⊡hb·cv	ⓣrad/⊡hb·lg	≈ $70.5556966669489_{\text{d}}$ ⓣrad/km
prime·quadcia·curvel	⊡q↓cvℓ	**prime·stadial·curvature**	⊡ς·cv	ⓣrad/⊡ς·lg	≈ $5.8796413889124_{\text{d}}$ ⓣrad/km
prime·pentcia·curvel	⊡p↓cvℓ	**prime·dromal·curvature**	⊡dr·cv	ⓣrad/⊡dr·lg	≈ $489.970115742701_{\text{d}}$ ⓣrad/Mm
prime·hexcia·curvel	⊡h↓cvℓ	**prime·itineral·curvature**	⊡itn·cv	ⓣrad/⊡itn·lg	≈ $40.8308429785584_{\text{d}}$ ⓣrad/Mm

Here, the choice of modifier in each colloquial name corresponds to the Primel length unit appearing in the denominator in the Derivation column.

Radiality and curvature characterize the circular path taken by a rotating object. The smaller the radiality, the greater the curvature, and thus the more angular displacement occurs per linear displacement. The greater the radiality, the smaller the curvature, and thus the less angular displacement occurs per linear displacement.

The prime·radiel and the prime·curvel turn out to be useful modifiers: When units of linear mechanics are multiplied by one or the other of these, they can be neatly transformed into analogous units of rotational mechanics.

Square Radiality, or Steradiality. The square of radiality is also a quantity of interest. The square of the prime·radiel is the **prime·squaradiel** (abbreviated ⊡sdℓ). A useful synonym for this, in the context of spherical geometry and solid angles, is the **prime·steradiel** (also abbreviated ⊡sdℓ). Note that this is equivalent to one prime·areanel per true·squaradian or true·steradian:

$$⊡\text{sd}\ell = ⊡\text{rd}\ell^2 = \frac{⊡\text{lg}\ell^2}{ⓣ\text{rad}^2} = \frac{⊡\text{ar}\ell}{ⓣ\text{sr}} = 67.2741710\overline{0694}_{\text{z}} \frac{\text{mm}^2}{ⓣ\text{sr}}$$

Scalings of this unit, along with their colloquial synonyms, include:

[10] Contraction of **prime·curvaturel**.

Unit	Abbrev	Colloquial	Abbrev	Derivation	SI Equivalent
prime·squaradiel **prime·steradiel**	⊡sdℓ	**prime·morsel·squaradiality** **prime·morsel·steradiality**	⊡mo·sd	⊡mo·ar/ⓣsr	$67.2741710069\bar{4}_d$ mm^2/ⓣsr
prime·biqua·squaradiel **prime·biqua·steradiel**	⊡b↑sdℓ	**prime·hand·squaradiality** **prime·hand·steradiality**	⊡hd·sd	⊡hd·ar/ⓣsr	96.87480625_d cm^2/ⓣsr
prime·quadqua·squaradiel **prime·quadqua·steradiel**	⊡q↑sdℓ	**prime·ell·squaradiality** **prime·ell·steradiality**	⊡ℓ·sd	⊡ℓ·ar/ⓣsr	1.39499721_d m^2/ⓣsr
prime·hexqua·squaradiel **prime·hexqua·steradiel**	⊡h↑sdℓ	**prime·habital·squaradiality** **prime·habital·steradiality**	⊡hb·sd	⊡hb·ar/ⓣsr	2.0087969824_d a/ⓣsr
prime·octqua·squaradiel **prime·octqua·steradiel**	⊡o↑sdℓ	**prime·stadial·squaradiality** **prime·stadial·steradiality**	⊡ς·sd	⊡ς·ar/ⓣsr	2.892666214656_d ha/ⓣsr
prime·decqua·squaradiel **prime·decqua·steradiel**	⊡d↑sdℓ	**prime·dromal·squaradiality** **prime·dromal·steradiality**	⊡dr·sd	⊡dr·ar/ⓣsr	4.16543934910464_d km^2/ⓣsr
prime·unnilqua·squaradiel **prime·unnilqua·steradiel**	⊡un↑sdℓ	**prime·itineral·squaradiality** **prime·itineral·steradiality**	⊡itn·sd	⊡itn·ar/ⓣsr	599.823266271068_d km^2/ⓣsr

Here, the choice of modifier in each colloquial name corresponds to the Primel area unit appearing in the numerator in the Derivation column.[11]

Square Curvature, or Stercurvature. The reciprocal of *squaradiality* is *squarecurvature*. Likewise, the reciprocal of *steradiality* is *stercurvature*; this is a synonym applicable to the description of solid angles.

The coherent unit of squarecurvature in Primel is the **prime·squarecurvel** (abbreviated ⊡scvℓ), the square of the prime·curvel;[12] this is also known as the **prime·stercurvel**[13] (same abbreviation). This is equivalent to a true·steradian per prime·areanel:

$$\boxdot\text{scv}\ell = \boxdot\text{cv}\ell^2 = \frac{\textcircled{t}\text{rad}^2}{\boxdot\text{lg}\ell^2} = \frac{\textcircled{t}\text{sr}}{\boxdot\text{ar}\ell} \approx 1.4864545858124_d\,\frac{\textcircled{t}\text{sr}}{\text{cm}^2}$$

Scalings of this unit, along with their colloquial synonyms, include:

Unit	Abbrev	Colloquial	Abbrev	Derivation	SI Equivalent
prime·squarecurvel **prime·stercurvel**	⊡scvℓ	**prime·morsel·squarecurvature** **prime·morsel·stercurvature**	⊡mo·scv	ⓣsr/⊡mo·ar	$\approx 1.4864545858124_d$ ⓣsr/mm^2
prime·bicia·squarecurvel **prime·bicia·stercurvel**	⊡b↓scvℓ	**prime·hand·squarecurvature** **prime·hand·stercurvature**	⊡hd·scv	ⓣsr/⊡hd·ar	$\approx 1.032260129132290_d$ ⓣsr/cm^2
prime·quadcia·squarecurvel **prime·quadcia·stercurvel**	⊡q↓scvℓ	**prime·ell·squarecurvature** **prime·ell·stercurvature**	⊡ℓ·scv	ⓣsr/⊡ℓ·ar	$\approx 0.71684731189742_d$ ⓣsr/m^2
prime·hexcia·squarecurvel **prime·hexcia·stercurvel**	⊡h↓scvℓ	**prime·habital·squarecurvature** **prime·habital·stercurvature**	⊡hb·scv	ⓣsr/⊡hb·ar	$\approx 0.497810633262099_d$ ⓣsr/a
prime·octcia·squarecurvel **prime·octcia·stercurvel**	⊡o↓scvℓ	**prime·stadial·squarecurvature** **prime·stadial·stercurvature**	⊡ς·scv	ⓣsr/⊡ς·ar	$\approx 0.345701828654235_d$ ⓣsr/ha
prime·deccia·squarecurvel **prime·deccia·stercurvel**	⊡d↓scvℓ	**prime·dromal·squarecurvature** **prime·dromal·stercurvature**	⊡dr·scv	ⓣsr/⊡dr·ar	$\approx 0.240070714343219_d$ ⓣsr/km^2
prime·unnilcia·squarecurvel **prime·unnilcia·stercurvel**	⊡un↓scvℓ	**prime·itineral·squarecurvature** **prime·itineral·stercurvature**	⊡itn·scv	ⓣsr/⊡itn·ar	$\approx 0.16671577384945_d$ ⓣsr/mym^2

Here, the choice of modifier in each colloquial name corresponds to the Primel area unit appearing in the denominator in the Derivation column.[14]

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[11] a = are = metric unit of area = square dekameter = 100_d square meters; ha = hectare = square hectometer = $10,000_d$ square meters

[12] Tom Pendlebury, in his Tim-Grafut-Maz (TGM) metrology (see https://dozenal.org/article-goodman-tgm-coherent-dozenal-metrology), introduced prefixes **rada**, **radi**, **quara**, **quari**, as TGM's own version of (respectively) **radiel**, **curvel**, **squaradiel**, **squarecurvel**. Except that Pendlebury's forms resemble his scaling prefixes, rather than independent unit names.

[13] Contraction of **prime·stercurvaturel**.

[14] mym = myriameter = $10{,}000_d$ meters = 10_d kilometers

Angular Displacement. The rotational analog for length or linear displacement is of course *angular displacement*. From the equation $\overset{\frown}{\theta} = s/\underset{\smile}{r}$ above, we see that true·angular displacement $\overset{\frown}{\theta}$ (as defined by Primel) is the ratio of arc length s to the radiality $\underset{\smile}{r}$ of the rotation.

Primel's coherent unit of angular displacement is the **prime·ang·lengthel** (abbreviated ⊡∡lgℓ).[15] This can be formulated as the **prime·curvel·lengthel** (abbreviated ⊡cvℓ·lgℓ).

$$\text{⊡∡lgℓ} = \text{⊡cvℓ·lgℓ} = \frac{\text{ⓣrad}}{\text{⊡lgℓ}} \cdot \text{⊡lgℓ} = \text{ⓣrad}$$

Of course this is just a synonym for the **true·radian**. (We can generate even more synonyms by substituting synonyms for **lengthel**, including **displacel**, **distancel**, etc.)

Angular Area. By a similar argument, the angular or spherical analog for planar area is *angular area*. Primel's coherent unit of angular area is the **prime·ang·areanel** (abbreviated ⊡∡arℓ). This can be formulated as the **prime·squarecurvel·areanel** (abbreviated ⊡scvℓ·arℓ).

$$\text{⊡∡arℓ} = \text{⊡scvℓ·arℓ} = \frac{\text{ⓣrad}^2}{\text{⊡lgℓ}^2} \cdot \text{⊡lgℓ}^2 = \text{ⓣrad}^2 = \text{ⓣsr}$$

Of course, this is just a synonym for the **true·squaradian** or **true·steradian**.

Angular Velocity. The rotational analog for linear velocity is *angular velocity*, which is the time rate of change of angular displacement. In the conventional SI approach, this is symbolized as ω, and is defined in terms of dimensionless angle θ:

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{r}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{v_\perp}{r} \qquad v_\perp = \frac{\mathrm{d}s}{\mathrm{d}t} \qquad \text{(SI)}$$

where $v_\perp$ is the tangential (linear) velocity. Note that, because θ is dimensionless, and because r is simply the radius (a length), this gives ω dimensionality indistinguishable from frequency (inverse time). This is problematic.

In contrast, Primel symbolizes true·angular velocity as $\overset{\frown}{\omega}$, defined in terms of true·angle $\overset{\frown}{\theta}$ and radiality $\underset{\smile}{r}$:

$$\overset{\frown}{\omega} = \frac{\mathrm{d}\overset{\frown}{\theta}}{\mathrm{d}t} = \frac{1}{\underset{\smile}{r}}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{v_\perp}{\underset{\smile}{r}} \qquad v_\perp = \frac{\mathrm{d}s}{\mathrm{d}t} \qquad \text{(Primel)}$$

This gives $\overset{\frown}{\omega}$ the dimensionality of true·angle per time.

Similar analysis applies when considering angular velocity as a vector in three dimensions:

$$\boldsymbol{\omega} = \frac{\mathbf{r} \times \mathbf{v}}{\|\mathbf{r}\|^2} \qquad \mathbf{v}_\perp = \boldsymbol{\omega} \times \mathbf{r} \qquad \text{(SI)}$$

$$\overset{\frown}{\boldsymbol{\omega}} = \frac{\underset{\smile}{\mathbf{r}} \times \mathbf{v}}{\|\underset{\smile}{\mathbf{r}}\|^2} \qquad \mathbf{v}_\perp = \overset{\frown}{\boldsymbol{\omega}} \times \underset{\smile}{\mathbf{r}} \qquad \text{(Primel)}$$

Primel's coherent unit of angular velocity is the **prime·ang·velocitel** (abbreviated ⊡∡vcℓ). This can be formulated as the **prime·curvel·velocitel** (abbreviated ⊡cvℓ·vcℓ). This simplifies to a true·radian per prime·timel:

$$\text{⊡∡vcℓ} = \text{⊡cvℓ·vcℓ} = \frac{\text{ⓣrad}}{\text{⊡lgℓ}} \cdot \frac{\text{⊡lgℓ}}{\text{⊡tmℓ}} = \frac{\text{ⓣrad}}{\text{⊡tmℓ}} = 34.56_{\mathrm{d}} \frac{\text{ⓣrad}}{\text{s}}$$

[15]Where **ang** is a contraction for **angular**, and is the pronunciation for the ∡ abbreviation.

Scalings[16] of this unit, along with their colloquial synonyms include:

Unit	Abbrev	Colloquial	Abbrev	Derivation	SI Equivalent
prime·ang·velocitel	⊡∡vcℓ	**prime·morsel·ang·velocity**	⊡mo∡vc	⊡mo·cv · ⊡vcℓ = ⓣrad/⊡vb·tm	34.56_{d} ⓣrad/s
prime·uncia·ang·velocitel	⊡u↓∡vcℓ	**prime·hand·ang·velocity**	⊡hd∡vc	⊡hd·cv · ⊡vcℓ = ⓣrad/⊡tw·tm	2.88_{d} ⓣrad/s
prime·bicia·ang·velocitel	⊡b↓∡vcℓ	**prime·ell·ang·velocity**	⊡ℓ∡vc	⊡ℓ·cv · ⊡vcℓ = ⓣrad/⊡lu·tm	14.4_{d} ⓣrad/min
prime·tricia·ang·velocitel	⊡t↓∡vcℓ	**prime·habital·ang·velocity**	⊡hb∡vc	⊡hb·cv · ⊡vcℓ = ⓣrad/⊡tr·tm	1.2_{d} ⓣrad/min
prime·quadcia·ang·velocitel	⊡q↓∡vcℓ	**prime·stadial·ang·velocity**	⊡ς∡vc	⊡ς·cv · ⊡vcℓ = ⓣrad/⊡br·tm	6_{d} ⓣrad/hr
prime·pentcia·ang·velocitel	⊡p↓∡vcℓ	**prime·dromal·ang·velocity**	⊡dr∡vc	⊡dr·cv · ⊡vcℓ = ⓣrad/⊡dw·tm	12_{d} ⓣrad/day
prime·hexcia·ang·velocitel	⊡h↓∡vcℓ	**prime·itineral·ang·velocity**	⊡itn∡vc	⊡itn·cv · ⊡vcℓ = ⓣrad/day	ⓣrad/day

Here, the choice of modifier in each colloquial name corresponds to the presumed Primel curvature unit used to convert the linear prime·velocitel unit into an angular velocity unit (shown in the Derivation column).

Angular Acceleration. The rotational analog of linear acceleration is *angular acceleration*. In the conventional SI approach, this is symbolized as α, which is the time rate of change of angular velocity ω:

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{r}\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = \frac{a_\perp}{r} \qquad a_\perp = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} \qquad \text{(SI)}$$

where $a_\perp$ is the tangential (linear) acceleration. Note that, because ω is indistinguishable from frequency, and because r is simply the radius, the dimensionality of α is indistinguishable from frequency squared (inverse time squared), which is problematic.

In contrast, Primel symbolizes true·angular acceleration as $\overset{\frown}{\alpha}$, defined in terms of true·angular velocity $\overset{\frown}{\omega}$ and radiality $\underset{\smile}{r}$:

$$\overset{\frown}{\alpha} = \frac{\mathrm{d}\overset{\frown}{\omega}}{\mathrm{d}t} = \frac{1}{\underset{\smile}{r}}\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = \frac{a_\perp}{\underset{\smile}{r}} \qquad a_\perp = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} \qquad \text{(Primel)}$$

This gives $\overset{\frown}{\alpha}$ the dimensionality of true·angle per time squared.

Similar analysis applies when considering angular acceleration as a vector in three dimensions:

$$\boldsymbol{\alpha} = \frac{\mathbf{r} \times \mathbf{a}}{\|\mathbf{r}\|^2} \qquad \mathbf{a}_\perp = \boldsymbol{\alpha} \times \mathbf{r} \qquad \text{(SI)}$$

$$\overset{\frown}{\boldsymbol{\alpha}} = \frac{\underset{\smile}{\mathbf{r}} \times \mathbf{a}}{\|\underset{\smile}{\mathbf{r}}\|^2} \qquad \mathbf{a}_\perp = \overset{\frown}{\boldsymbol{\alpha}} \times \underset{\smile}{\mathbf{r}} \qquad \text{(Primel)}$$

Primel's coherent unit of true·angular acceleration is the **prime·ang·accelerel** (abbreviated ⊡∡accℓ). This can be formulated as the **prime·curvel·accelerel** (abbreviated ⊡cvℓ·accℓ). This is equivalent to one true·radian per prime·timel squared:

$$⊡∡\text{acc}\ell = ⊡\text{cv}\ell{\cdot}\text{acc}\ell = \frac{ⓣ\text{rad}}{⊡\text{lg}\ell} \cdot \frac{⊡\text{lg}\ell}{⊡\text{tm}\ell^2} = \frac{ⓣ\text{rad}}{⊡\text{tm}\ell^2} = 1194.3936_{\mathrm{d}}\,\frac{ⓣ\text{rad}}{\text{s}^2}$$

Scalings of this unit, along with their colloquial synonyms, include:

[16] tm = time, vb = vibe, tw = twinkling, lu = lull, tr = trice, br = breather, dw = dwell.

Unit	Abbrev	Colloquial	Abbrev	Derivation		SI Equivalent
prime·ang·accelerel	⊡∡accℓ	**prime·morsel·ang·acceleration**	⊡mo∡acc	⊡mo·cv · ⊡accℓ	$=$ ⓣrad/⊡vb·tm^2	1194.3936_{d} ⓣrad/s^2
prime·uncia·ang·accelerel	⊡u↓∡accℓ	**prime·hand·ang·acceleration**	⊡hd∡vc	⊡hd·cv · ⊡accℓ	$= 10_{\text{z}}$ ⓣrad/⊡tw·tm^2	99.5328_{d} ⓣrad/s^2
prime·bicia·ang·accelerel	⊡b↓∡accℓ	**prime·ell·ang·acceleration**	⊡ℓ∡acc	⊡ℓ·cv · ⊡accℓ	$=$ ⓣrad/⊡tw·tm^2	8.2944_{d} ⓣrad/s^2
prime·tricia·ang·accelerel	⊡t↓∡accℓ	**prime·habital·ang·acceleration**	⊡hb∡acc	⊡hb·cv · ⊡accℓ	$= 10_{\text{z}}$ ⓣrad/⊡lu·tm^2	0.6912_{d} ⓣrad/s^2
prime·quadcia·ang·accelerel	⊡q↓∡accℓ	**prime·stadial·ang·acceleration**	⊡ς∡acc	⊡ς·cv · ⊡accℓ	$=$ ⓣrad/⊡lu·tm^2	0.0576_{d} ⓣrad/s^2
prime·pentcia·ang·accelerel	⊡p↓∡accℓ	**prime·dromal·ang·acceleration**	⊡dr∡acc	⊡dr·cv · ⊡accℓ	$= 10_{\text{z}}$ ⓣrad/⊡tr·tm^2	0.0048_{d} ⓣrad/s^2
prime·hexcia·ang·accelerel	⊡h↓∡accℓ	**prime·itineral·ang·acceleration**	⊡itn∡acc	⊡itn·cv · ⊡accℓ	$=$ ⓣrad/⊡tr·tm^2	0.0004_{d} ⓣrad/s^2

Here, the choice of modifier in each colloquial name corresponds to the presumed Primel curvature unit used to convert the linear prime·accelerel unit into an angular velocity unit (shown in the Derivation column).

Angular Mass or Moment of Inertia. The rotational analog of mass is known as *moment of inertia* or *angular mass*. SI defines this as the mass m of the rotating object times the square of the radius r. But Primel uses the radiality $\underset{\smile}{r}$ instead:

$$I = mr^2 \qquad \text{(SI)} \qquad\qquad \underset{\smile}{I} = m\underset{\smile}{r}^2 \qquad \text{(Primel)}$$

Thus Primel's definition of true·angular mass is actually equivalent to SI's version with the true·radianic operator applied *twice*.

Primel's coherent unit of angular mass is the **prime·ang·massel** (abbreviated ⊡∡msℓ). This can be formulated as the **prime·squaradiel·massel** (abbreviated ⊡sdℓ·msℓ):

$$⊡∡\text{ms}\ell = ⊡\text{sd}\ell\cdot\text{ms}\ell = \frac{⊡\text{lg}\ell^2}{ⓣ\text{rad}^2} \cdot ⊡\text{ms}\ell \approx 0.371200648827305_{\text{d}}\ \frac{\text{g}\cdot\text{cm}^2}{\text{rad}^2}$$

Scalings of this unit, along with their colloquial synonyms, include:

Unit	Abbrev	Colloquial	Abbrev	Derivation		SI Equivalent
prime·ang·massel	⊡∡msℓ	**prime·morsel·ang·mass**	⊡mo∡ms	⊡mo·sd · ⊡mo·ms	$=$ ⊡rdℓ^2 · ⊡msℓ	$\approx 0.371200648827305_{\text{d}}\ \frac{\text{g}\cdot\text{cm}^2}{ⓣ\text{rad}^2}$
prime·pentqua·ang·massel	⊡p↑∡msℓ	**prime·hand·ang·mass**	⊡hd∡ms	⊡hd·sd · ⊡hd·ms	$=$ ⊡u↑rdℓ^2 · ⊡t↑msℓ	$\approx 0.923665998489965_{\text{d}}\ \frac{\text{kg}\cdot\text{dm}^2}{ⓣ\text{rad}^2}$
prime·decqua·ang·massel	⊡d↑∡msℓ	**prime·ell·ang·mass**	⊡ℓ∡ms	⊡ℓ·sd · ⊡ℓ·ms	$=$ ⊡b↑rdℓ^2 · ⊡h↑msℓ	$\approx 2.29837657736254_{\text{d}}\ \frac{\text{kg}\cdot\text{m}^2}{ⓣ\text{rad}^2}$

Here, the choice of modifier in each colloquial name corresponds to the presumed Primel squaradiality unit (or Primel reciprocal squaradiality unit) used to convert a linear Primel mass unit into an angular mass unit (shown in the Derivation column). Each of these steps increases the magnitude by 5 powers of dozen because they involve one power of dozen, squared, from the square radiality unit, plus three powers of dozen from the mass unit.

Angular Momentum. The rotational analog of momentum is known as *angular momentum*. SI symbolizes this as a vector $\mathbf{L}$ and calculates it as the cross-product of the radius vector $\mathbf{r}$ (position vector of the rotating object relative to the axis of rotation) times the linear momentum vector $\mathbf{p}$. SI can also calculate it as the product of its version of angular mass I multiplied by its version of angular velocity vector $\boldsymbol{\omega}$. This is by direct analogy with linear momentum being the product of mass m and linear velocity vector $\mathbf{v}$:

$$\mathbf{L} = I\boldsymbol{\omega} = \mathbf{r} \times \mathbf{p} \qquad \mathbf{p} = m\mathbf{v} \qquad \text{(SI)}$$

However, this gives $\mathbf{L}$ a problematic dimensionality indistinguishable from action (which is linear momentum times linear displacement).

In contrast, Primel symbolizes its version of angular momentum as a vector $\underset{\smile}{\mathbf{L}}$ and calculates it as the cross product of the *radiality* vector $\underset{\smile}{\mathbf{r}}$ of the rotating object multiplied by its linear momentum vector $\mathbf{p}$. Primel can also calculate it as the product of its true·angular mass $\underset{\smile\smile}{I}$, times its true·angular velocity vector $\overset{\frown}{\boldsymbol{\omega}}$:

$$\underset{\smile}{\mathbf{L}} = \underset{\smile\smile}{I}\overset{\frown}{\boldsymbol{\omega}} = \underset{\smile}{\mathbf{r}} \times \mathbf{p} \qquad \mathbf{p} = m\mathbf{v} \qquad \text{(Primel)}$$

This gives $\underset{\smile}{\mathbf{L}}$ a unique dimensionality equivalent to action per true·angular displacement, analogous to linear momentum being equivalent to action per linear displacement.

This distinction between action and angular momentum is at the heart of why the Planck constant seems to have two values: h versus $\hbar = h/\tau$. Both of these are expressed in units of action, but in reality physicists should be using $\underset{\smile}{\hbar}$:

$$\underset{\smile}{\hbar} = \frac{\hbar}{\text{ⓣrad}} = \frac{h/\tau}{\text{ⓣrad}} = \frac{h}{\tau \cdot \text{ⓣrad}} = \frac{h}{\text{ⓣ}\odot}$$

In other words, when we include the missing true·angular units, both h and $\hbar$ express the *same* constant: $\underset{\smile}{\hbar}$, the quantum of true·angular momentum. The only difference is whether this constant is expressed in action units per true·radian or per true·turn.

Primel's coherent unit of true·angular momentum is the **prime·ang·momentumel**[17] (abbreviated ⊡∡mmℓ). This can be formulated as the **prime·radiel·momentumel** (abbreviated ⊡rdℓ·mmℓ). Note that the result is equivalent to a primel·actionel per true·radian:

$$\text{⊡∡mmℓ} = \text{⊡rdℓ·mmℓ} = \frac{\text{⊡lgℓ}}{\text{ⓣrad}} \cdot \text{⊡mmℓ} = \frac{\text{⊡actℓ}}{\text{ⓣrad}} \approx 12.8286944234717_{\text{d}} \frac{\text{g·cm}^2/\text{s}}{\text{ⓣrad}}$$

This can also be formulated as the product of a primel·ang·massel times a primel·ang·velocitel:

$$\text{⊡∡mmℓ} = \text{⊡∡msℓ} \cdot \text{⊡∡vcℓ} = \text{⊡sdℓ·msℓ} \cdot \text{⊡cvℓ·vcℓ} = \left(\text{⊡rdℓ}^2 \cdot \text{⊡msℓ}\right) \cdot \left(\text{⊡rdℓ}^{-1} \cdot \text{⊡vcℓ}\right) = \text{⊡rdℓ·mmℓ}$$

Scalings of this unit, along with their colloquial synonyms, include:

Unit	Abbrev	Colloquial	Abbrev	Derivation	SI Equivalent
prime·ang·momentumel	⊡∡mmℓ	**prime·morsel·ang·momentum**	⊡mo∡mm	⊡mo·rd · ⊡mo·mm	$\approx 12.8286944234717_{\text{d}}\ \frac{\text{g·cm}^2/\text{s}}{\text{ⓣrad}}$
prime·pentqua·ang·momentumel	⊡q↑∡mmℓ	**prime·hand·ang·momentum**	⊡hd∡mm	⊡hd·rd · ⊡hd·mm	$\approx 0.923665998489965_{\text{d}}\ \frac{\text{kg·dm}^2/\text{s}}{\text{ⓣrad}}$
prime·decqua·ang·momentumel	⊡o↑∡mmℓ	**prime·ell·ang·momentum**	⊡ℓ∡mm	⊡ℓ·rd · ⊡ℓ·mm	$\approx 2.29837657736254_{\text{d}}\ \frac{\text{Mg·m}^2/\text{s}}{\text{ⓣrad}^2}$

Here, the choice of modifier in each colloquial name corresponds to both the Primel radiality unit and the linear Primel momentum unit (shown in the Derivation column). Each of these steps increases the magnitude by 4 powers of dozen because they involve one power of dozen from the radiality unit, plus three powers of dozen from the momentum unit.

Angular Force. The rotational analog of force is known as *angular force* or *torque*. SI symbolizes this as a vector $\mathbf{T}$,[18] and calculates it as the cross-product of the radius vector $\mathbf{r}$ (position vector of the rotating object relative to the axis of rotation) times the linear force vector $\mathbf{F}$. SI can also calculate it as the product of its version of angular mass I multiplied by its version of angular acceleration vector $\boldsymbol{\alpha}$. This is by direct analogy with linear force being the product of mass m and linear acceleration vector $\mathbf{a}$:

$$\mathbf{T} = I\boldsymbol{\alpha} = \mathbf{r} \times \mathbf{F} \qquad \mathbf{F} = m\mathbf{a} \qquad \text{(SI)}$$

[17] An acceptable contraction for **momentumel** is **momel**.

[18] This is uppercase Greek tau, indistinguishable from Latin T. SI actually uses lowercase tau ($\boldsymbol{\tau}$) for this. But I am avoiding this in order to reserve $\tau = 2\pi$ as a circle constant.

However, this gives $\mathbf{T}$ a problematic dimensionality indistinguishable from work (which is linear force times linear displacement).

In contrast, Primel symbolizes its version of true·angular force as vector $\underset{\smile}{\mathbf{T}}$, and calculates this as the cross product of the *radiality* vector $\underset{\smile}{\mathbf{r}}$ times its linear force vector $\mathbf{F}$. Primel can also calculate it as the product of its version of true·angular mass $\underset{\smile}{\underset{\smile}{I}}$, times its version of true·angular acceleration vector $\overset{\frown}{\boldsymbol{\alpha}}$:

$$\underset{\smile}{\mathbf{T}} = \underset{\smile}{\underset{\smile}{I}}\,\overset{\frown}{\boldsymbol{\alpha}} = \underset{\smile}{\mathbf{r}} \times \mathbf{F} \qquad \mathbf{F} = m\mathbf{a} \qquad \text{(Primel)}$$

This gives $\underset{\smile}{\mathbf{T}}$ its own unique dimensionality equivalent to work (i.e., energy) per angular displacement, analogous to linear force being equivalent to work per linear displacement.

Primel's coherent unit of true·angular force is the **prime·ang·forcel** (abbreviated ⊡∡fcℓ). This is formulated as the **prime·radiel·forcel** (abbreviated ⊡rdℓ·fcℓ), which is equivalent to one prime·workel (or prime·energel) per true·radian:

$$\boxdot\measuredangle\text{fc}\ell = \boxdot\text{rd}\ell\cdot\text{fc}\ell = \frac{\boxdot\text{lg}\ell}{\text{ⓣrad}}\cdot\boxdot\text{fc}\ell = \frac{\boxdot\text{wk}\ell}{\text{ⓣrad}} \approx 44.3359679275181\,\frac{\mu\text{J}}{\text{ⓣrad}}$$

This can also be computed as a prime·ang·massel times a prime·ang·accelerel:

$$\boxdot\measuredangle\text{fc}\ell = \boxdot\measuredangle\text{ms}\ell\cdot\boxdot\measuredangle\text{acc}\ell = \boxdot\text{sd}\ell\cdot\text{ms}\ell\cdot\boxdot\text{cv}\ell\cdot\text{acc}\ell = \left(\boxdot\text{rd}\ell^{2}\cdot\boxdot\text{ms}\ell\right)\cdot\left(\boxdot\text{rd}\ell^{-1}\cdot\boxdot\text{acc}\ell\right) = \boxdot\text{rd}\ell\cdot\text{fc}\ell$$

Scalings of this unit, along with their colloquial synonyms, include:

Unit	Abbrev	Colloquial	Abbrev	Derivation	SI Equivalent
prime·ang·forcel	⊡∡fcℓ	**prime·morsel·ang·force**	⊡mo∡fc	⊡mo·rd · ⊡mo·fc = ⊡rdℓ · ⊡fcℓ	$\approx 44.3359679275181_{\text{d}}\ \frac{\mu\text{J}}{\text{ⓣrad}}$
prime·quadqua·ang·forcel	⊡q↑∡fcℓ	**prime·hand·ang·force**	⊡hd∡fc	⊡hd·rd · ⊡hd·fc = ⊡u↑rdℓ · ⊡t↑fcℓ	$\approx 0.919350630945015_{\text{d}}\ \frac{\text{J}}{\text{ⓣrad}}$
prime·octqua·ang·forcel	⊡o↑∡fcℓ	**prime·ell·ang·force**	⊡ℓ∡fc	⊡ℓ·rd · ⊡ℓ·fc = ⊡b↑rdℓ · ⊡h↑fcℓ	$\approx 2.29837657736254_{\text{d}}\ \frac{\text{kJ}}{\text{ⓣrad}}$

Here, the choice of modifier in each colloquial name corresponds to both the Primel radiality unit and the linear Primel force unit (shown in the Derivation column). Each of these steps increases the magnitude by 4 powers of dozen because they involve one power of dozen from the radiality unit, plus three powers of dozen from the force unit.

Translational and Rotational Kinetic Energy. Primel's versions of true·angular mass ($\underset{\smile}{\underset{\smile}{I}}$) and true·angular velocity ($\overset{\frown}{\omega}$) can be used to calculate the rotational component of an object's kinetic energy (E_{R}). This is directly analogous to how the translational component of kinetic energy (E_{T}) can be calculated from the object's mass (m) and linear velocity (v):

$$E_{\text{R}} = \frac{1}{2}\underset{\smile}{\underset{\smile}{I}}\,\overset{\frown}{\omega}^{2} \qquad E_{\text{T}} = \frac{1}{2}mv^{2}$$

Note here that two true·radianic operators cancel out two true·radian operators. We can confirm this by looking at Primel's units for these equalities:

$$\boxdot\text{ng}\ell = \boxdot\measuredangle\text{ms}\ell\cdot\boxdot\measuredangle\text{vc}\ell^{2} = \boxdot\text{sd}\ell\cdot\text{ms}\ell\cdot\boxdot\text{cv}\ell\cdot\text{vc}\ell^{2} = \left(\boxdot\text{rd}\ell^{2}\cdot\boxdot\text{ms}\ell\right)\left(\boxdot\text{rd}\ell^{-2}\cdot\boxdot\text{vc}\ell^{2}\right) = \boxdot\text{ms}\ell\cdot\boxdot\text{vc}\ell^{2} = \boxdot\text{ng}\ell$$

Reusability of Unit Names. All of the quantitel and quantitelic unit names described above have been specific to the Primel metrology, so they have all sported Primel's brand prefix **prime·** or its brand mark ⊡. Nevertheless, they can be reused for other metrologies by simply replacing Primel's brand mark with the brand mark of another metrology (and of course recomputing the unit sizes in terms of that metrology's own "mundane realities").

Angular Transcendental Functions

An additional problem encountered when considering the dimensionality of angular displacement is the question of how to treat trigonometric and other transcendental functions that take angular displacements as arguments. Such functions often have Taylor series expansions, which are infinite sums of terms each containing differing powers of the argument. Such terms would not be commensurate with each other, and so could not be added together, unless the original argument were dimensionless. To see the solution to this issue, let us first consider how to apply the true·radian and true·radianic operators to a mathematical function.

The true·radian operator applied to a function f is fairly straightforward. Function f would take some number of arguments but return a result that can be interpreted as an dimensionless angle. Without loss of generality, let's consider a function f taking one argument x:

$$f(x) = \theta$$

Applying the true·radian operator defines a new function $\overset{\frown}{f}$ that converts the result of f to true·radians:

$$\overset{\frown}{f}(x) = f(x) \cdot \textcircled{t}\text{rad} = \overset{\frown}{\theta}$$

The true·radianic operator applied to a function is more interesting. In this case, the original function f would take arguments, some of which may be dimensionless angles. Without loss of generality, let's consider a unary function $f(\theta)$ where the argument θ is a dimensionless angle. Applying the true·radianic operator defines a new function $\underset{\smile}{f}$ that would take a true·angle argument $\overset{\frown}{\theta}$. What this function would do is apply the true·radianic operator to its argument $\overset{\frown}{\theta}$, effectively canceling out the true·radian operator on the argument, turning it into dimensionless θ, before passing it on to the original function f:

$$\underset{\smile}{f}\left(\overset{\frown}{\theta}\right) = f\left(\underset{\smile}{\overset{\frown}{\theta}}\right) = f\left(\theta\right)$$

In this way, this introduces a new "complete" function $\underset{\smile}{f}$ accepting a true·angular quantity $\overset{\frown}{\theta}$. But since $\underset{\smile}{f}$ extracts the dimensionless measure quantity $\underset{\smile}{\overset{\frown}{\theta}} = \theta$ and passes that on to the original function f, this leaves f itself free to calculate any terms it likes from θ. For instance, if f is transcendental, then it is free to do an infinite Taylor series expansion:

$$\underset{\smile}{\sin}\,\overset{\frown}{\theta} = \sin \underset{\smile}{\overset{\frown}{\theta}} = \sin\theta = \sum_{n=0}^{\infty} (-1)^n \, \frac{\theta^{2n+1}}{(2n+1)!} = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$$

$$\underset{\smile}{\cos}\,\overset{\frown}{\theta} = \cos \underset{\smile}{\overset{\frown}{\theta}} = \cos\theta = \sum_{n=0}^{\infty} (-1)^n \, \frac{\theta^{2n}}{(2n)!} \quad = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$$

$$\underset{\smile}{\text{expi}}\,\overset{\frown}{\theta} = \text{expi}\,\underset{\smile}{\overset{\frown}{\theta}} = \text{expi}\,\theta = e^{i\theta} = \sum_{n=0}^{\infty} i^n \frac{\theta^n}{n!} = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} - \ldots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots\right) = \cos\theta + i\sin\theta = \underset{\smile}{\cos}\,\overset{\frown}{\theta} + i\,\underset{\smile}{\sin}\,\overset{\frown}{\theta}$$

This is why it was important to distinguish $\overset{\frown}{\theta}$ as a true physical quantity with physical dimension, while nevertheless preserving the mathematical status-quo interpretation of θ as a dimensionless pure number.

The heart of the matter is that mathematical functions, including transcendental functions like sin, cos and expi, actually compute relationships between pure mathematical abstractions with no regard to any sort of physical manifestation in the real world. Not even the geometric interpretation as angles in the plane is really pertinent to the abstraction (although it can provide intuition to aid in understanding it).

On the other hand, functions such as sin, cos, and expi *do* relate to real physical quantities that can actually be measured. But to further manipulate these quantities, we need to strip them of their physical dimension and bridge into the world of pure mathematics. This is fine, but it does mean that we have identified two quite different, yet related, kinds of function, applicable to very different contexts. Problems only arise when we try to conflate the two.

More to Come

Designing Primel's angular mechanics units has required diverging from SI's approach to the subject. But as it turns out, such divergence was not entirely unprecedented.

The next article in this series will cover Primel's units for electricity and magnetism. You may find the divergence from SI's treatment of this subject even more dramatic than angular mechanics. I will examine not only the usual types of electromagnetic quantities encountered by first-year physics and engineering students, but in fact all the various kinds of quantities embodied in famous equations by James Clerk Maxwell and others.[19] Primel's approach does not in any way change the dimensionality of these quantities. However, in order to derive a balanced system of quantitel unit names for them, it does make interesting changes to the *terminology* applied to electromagnetic quantities. This may actually prove to be the most controversial aspect of the metrology. I hope you all get a *charge* out of it.☺ ⁙

Primel ⊡ Summary of Angular Mechanical Units				
Quantity	Quantitel	Abbrev	Decomposition	Metric Equivalents
Plane Angle Angular Length	**true·radian** **prime·ang·lengthel**	ⓣrad ⊡∡lgℓ	ⓣrad	ⓣrad
Square Angle Solid Angle Angular Area	**true·squaradian** **true·steradian** **prime·ang·areanel**	ⓣsr ⊡∡arℓ	ⓣrad^2	ⓣrad^2 ⓣsr
Radiality	**prime·radiel**	⊡∡rdℓ	⊡lgℓ/ⓣrad	$= 8.20208\overline{3}_{\text{d}}$ mm/rad
Curvature	**prime·curvel**	⊡∡cvℓ	ⓣrad/⊡lgℓ	$\approx 1.219202438404877_{\text{d}}$ ⓣrad/cm
Squaradiality Steradiality	**prime·squaradiel** **prime·steradiel**	⊡∡sdℓ	⊡arℓ/ⓣsr	$= 67.2741710069\overline{4}_{\text{d}}$ mm^2/ⓣrad^2
Squarecurvature Stercurvature	**prime·squarecurvel** **prime·stercurvel**	⊡∡scvℓ	ⓣsr/⊡arℓ	$\approx 1.4864545858124_{\text{d}}$ ⓣrad^2/cm^2
Angular Velocity	**prime·ang·velocitel**	⊡∡vcℓ	ⓣrad/⊡tmℓ	$= 34.56_{\text{d}}$ ⓣrad/s
Angular Acceleration	**prime·ang·accelerel**	⊡∡accℓ	ⓣrad/⊡$\text{tm}\ell^2$	$= 1194.3936_{\text{d}}$ ⓣrad/s^2
Angular Mass Moment of Inertia	**prime·ang·massel**	⊡∡msℓ	⊡msℓ·⊡$\text{lg}\ell^2$/ⓣrad^2	$\approx 0.371200648827305_{\text{d}}$ g·cm^2/ⓣrad^2
Angular Momentum	**prime·ang·momentumel**	⊡∡mmℓ	⊡actℓ/ⓣrad	$\approx 12.8286944234717_{\text{d}}$ g·cm^2/s/ⓣrad
Angular Force Torque	**prime·ang·forcel**	⊡∡fcℓ	⊡wkℓ/ⓣrad	$\approx 44.3359679275181_{\text{d}}$ µJ/ⓣrad

[19] See https://en.wikipedia.org/wiki/Maxwell's_equations.