The Dozenal Society of America

A Rational Solution to the Problem of Weights and Measures[*]

Sidney A. Reeve

1. The question of what system of weights and measures most fully meets the demands of the technical world has recently received renewed discussion. The source of this fresh impetus has lain in the projection of a national law, the adoption of which would render the universal use of the metric system compulsory. It is not the object of this paper to undertake discussion of the merits of this bill; it is to present briefly some of the advantages to be gained from advance in another direction. This alternate line of progress has already been ably advocated, at one time or another, in a general way; but the concrete programme for procedure which is herein presented has never yet been suggested, so far as the writer is aware.

2. As a preliminary step some of the fundamental attributes of the metric and the English systems will be outlined as the writer sees them. In doing this no attempt will be made to conceal the firm opinion that the metric system is not naturally and inherently adapted to industrial needs, and that to commit ourselves finally to its universal, compulsory adoption would be a mistake of immeasurable magnitude. But the writer also disclaims any belief that the metric system has been proven by experience incapable of adoption in engineering and industrial works. Locally and occasionally it has been so adopted. There has been no resultant catastrophe. There has not always been even resultant rejection. But it is broadly and plainly true that the resultant gains have not been sufficient to spread appreciably the field of experiment with the metric system—not even so rapidly as industrial effort has extended. For thirty-six years the use of the metric system has been open to all who cared to try it. In all those years the proportion of those who did to the whole number who might has not perceptibly increased.

3. The reasons for both the continued advocacy and the continued rejection of the metric system are plain. They are parallel and quite compatible.

(a) The metric system is attractive because its measures are arranged on the same system as our numerical notation.

(b) The metric system is cumbrous because it is decimal in its arrangement.

4. To state that the advantages of the metric system lie in the fact that its arrangement is decimal is erroneous and deceptive. Had our numerical system been based upon the octonal or the duodecimal plan, the scientific originators of the metric system would have adopted just as promptly and have urged just as vehemently a system of weights and measures also based upon the octonal or duodecimal plan. If that had been done the then system would have every advantage now offered by

[*]Originally published in Transactions of the American Society of Mechanical Engineers, Vol. XXIV (New York: 1127 (1903.)), 184.–215., and presented at the New York meeting of that Society in December 1126. It is made available in this newly typeset, standalone format by the Dozenal Society of America, 11Ɛ↊ (2014.) (http://www.dozenal.org). Text courtesy of Google Books.

the metric. In addition it would have many more.

5. To support this last statement properly would be to duplicate much which has already been written, more ably than could be reproduced here, and which is accessible to every reader of these lines. In particular, in vol. xlix. of the *Popular Science Monthly*, will be found a full and cogent statement by Mr. Herbert Spencer of the reasons why the metric system has not found wide adoption. In the same volume is a counter-defence by one of the ablest of the advocates of the metric system, Dr. T. C. Mendenhall. No engineer may presume to a worthy opinion upon the weights and measures question without acquaintance with these writings.*

6. To summarize very briefly, Mr. Spencer's position is:

(1) That the natural evolution of systems of measures by popular adoption or rejection, or by the survival of the fittest, has ever been away from decimal divisions and toward the repeated division of a unit by twos and by threes.

(2) That this tendency has been only very slightly affected by the parallel presence of decimal systems of division, even when made compulsory by law.

7. Thus, in this country a decimal division of currency has been compulsory by law for over a century, and is backed by all the inconveniences involved in the departure of money-division from the standard system of notation, which is decimal. Yet the division of the standard unit, the dollar, by other factors than those of ten and its powers, by factors of two, three, four, six, eight, twelve, and sixteen, is practically universal. Of the three decimal divisions of the dollar; the dime, the cent, and the mill, the first and the last are unheard of as units of price in ordinary retail business; the other, the cent, is almost as apt to be split by a vulgar fraction as it is to be used in its integral purity. Of all our coins the favorites are the "half" and the "quarter." The dime is used much more to make change for the quarter, because five nickels are too cumbrous, than it is as one-tenth of a dollar. We could not get along without the half-dime, or "nickel." The cent is scarcely ever used to make change for a dime. In short, decimal subdivisions are much too far apart.

Even in the choice of rates of interest, where the burden of calculation is a maximum in proportion to the coin actually handled, there is little disposition to retain the decimal system. Fractional portions of units per cent. are not often stated as tenths, but more commonly as halves or quarters or eighths.

This is the final, present result of a century's experiment with a decimal system supported by legally compulsory adoption. In other countries and in other lines of measure than the monetary the experience is parallel. In short, all the advantages of having a system of measures upon the same basis as the system of notation are not sufficient to countervail the disadvantages of conducting the day's work upon any other basis than division by twos and threes.

8. Even in scientific work the same trouble is found. So long as instruments, scales, etc., are divided on the decimal system it is of course easiest to read them so. But when that artificial constraint is exceeded the natural basis for either estimating or assigning divisions is by twos or by threes. Every student has to be arbitrarily taught to estimate to tenths, and even then the result is inaccurate. Every intelligent young observer, on the other hand, naturally estimates well to halves, thirds, and quarters. In my own work, although I carefully instruct at the start against using numerical statements to a greater degree of accuracy than is naturally possible, yet I sanction and believe in observations made and stated in estimated divisions such as 0.25, 0.33, etc. For

*It is assumed that all readers of this paper are already acquainted with the earlier discussions of the metric system before this Society, to which no concrete reference is made.

the observer to estimate to tenths is difficult and inaccurate; to attempt to estimate to hundredths is absurd. Yet I more highly esteem the accuracy of such estimated divisions as those above stated than I do stated estimates of 0.2, 0.3, 0.4, etc.

9. It is only because scientific work involves so large a proportion of computation to a given amount of mensuration that the metric system is popular among scientists. For pure mensuration nothing will ever be able to compete successfully with the two-foot rule, with "two pints make one quart, four quarts make one gallon," with "twelve units make one dozen, twelve dozens make one gross," etc.*

10. But even for computative, scientific purposes a duodecimal system of both measures and numbers is infinitely to be preferred to a decimal system of both measures and numbers. The reasons are these:

(I) For *Mensuration* the advantages are as just stated.

(II) For *Computation:* (a) The mental burden involved in carrying in the head a duodecimal multiplication table, and in performing with it the simplest arithmetical processes, is much less than with the decimal system. One has only to faithfully try this experiment to be convinced.

(b) The degree of accuracy of a given number of significant digits is much enhanced. Four duodecimal digits possess twice the accuracy of four decimal digits; six possess three times the accuracy; nine possess five times the accuracy. This means twice the accuracy, for a given effort, in all engineering calculations, and from three to five times the accuracy in geodetic, astronomical, and physical work making use of logarithms.

11. The industrial and commercial world has already emphatically pronounced in favor of division by twos and threes, and is daily voicing its corroboration of this opinion. The scientific world has emphatically pronounced itself, not in favor of division by tenths, but of harmony between mensuration and notation. Confronted by these two facts the discussion can turn upon only one pivot, viz.:—Shall the industrial and commercial world give up (in adopting the metric system) what it has shown that it cannot be forced, even by law, to do without, for the sake of granting to the scientific world what the latter very feebly desires? Shall the scientific world obtain its desired harmony between measures and notation (in the metric system enforced by law) by saddling upon the industrial world another system on top of and by the side of the one which it will not, cannot abandon? Or shall the scientific world gain its desired harmony between measures and notation, and at the same time gain much added facility, by conforming its notation to the duodecimal system of measures upon which the commercial world necessarily conducts its daily transactions?

12. It has been urged that the industrial world cannot change to the metric system because of enormous loss of investment in tools, etc. It is to be said in reply to this, with truth and force, that no mere value of investment, even if it be greater than the metric advocates urge that it is, can properly constrain so momentous a decision. But the question goes deeper than that. It is also true, on the other hand, that not even the boldest disregard of expense can hope to alter the inherent human

*Dr. Mendenhall's reply to this, by quoting:

$5\frac{1}{2}$ yards	are	1 pole.
40 poles	"	1 furlong.
28 pounds	"	1 quarter.
$30\frac{1}{4}$ square yards	"	1 square pole
40 square poles	"	1 rood, etc.,

is irrelevant. Americans make no appreciable use of these units. Not one person in a thousand knows these tables.

preference for halves, thirds, and quarters over tenths; and no mere monetary gain, however imaginably great, could counterbalance the loss of human efficiency due to the repression of that preference, could it be accomplished.

13. On the other hand, the scientific world could change over from a decimal to a duodecimal system with greater ease than could any other portion of the human race make a similar change. Because

(I) It is intellectually the most flexible. I have tried the experiment of learning the duodecimal multiplication tables and of temporarily relying upon them and upon duodecimal notation for all computation. In spite of the inevitably frequent and disconcerting contact with the decimal system, in three days' time duodecimals were easier than decimals. I insist that it is easier to think in dozens than it is in tens. The tables for 2, 3, 4, 6, 8, and 9 in the duodecimal system are simplicity itself; only those for 2 and 5 in the decimal system can be compared with them. The obscure tables, where memorization alone can be of service, are 5 and 7 for the duodecimal system; the corresponding ones in the decimal system are 3, 4, 6, and 7, or twice as many. The tables for 8 and 9 in the decimal system and those for 10 and 11 (single digits) in the duodecimal occupy an intermediate position as to difficulty.

(II) The cost of replacing decimal tables, graduations, etc., in observatories and laboratories with duodecimal ones would be no greater, if nearly so great, as that of altering industrial tools, graduations, and tables from the present octonal or duodecimal to the decimal (metric) sytsem. When it is remembered that all astronomical work involves the cumbrous 60:1 division, for both arcs and for time, it is debatable whether, were the duodecimal logarithmic and other tables once in existence, observers would not find it worth while to translate observations from instruments where the graduations remained decimal into duodecimal records before computation, rather than to compute them in decimals.

14. The question, I repeat, is not one of possibility of change of systems, or of the cost of change. To avoid some change from the present intolerable confusion is impossible. On the other hand, the cost of any change whatever, commensurate with the needs of the situation, will be incalculable, in absolute units, and becomes greater each day. Only as a comparison between alternate methods can discussion of costs and gains be intelligent. Taken up in this way such discussion can lead to no other result than the choice of a duodecimal system of weights and measures harmonious with a duodecimal system of arithmetical notation.

15. There might be many such systems, any one of which would be better than any decimal system. To render the proposition concrete, however, the following suggestion of an outline for a system is offered.

Duodecimal Numbers.

The digits are to be those in use at present: 1, 2, 3, 4, 5, 6, 7, 8, and 9, with the addition of two new ones:

δ,	or *dek*,	having the value of decimal 10,	and
Ɛ,	or *eln*,	having the value of decimal 11.	

Both of these characters can be readily and rapidly made with the pencil, in a form not to be confused with any of the other digits when carelessly made. The name *dek* is drawn from the Latin *decem*; *eln* is an abbreviation of eleven. The form of dek (δ) recalls the idea of the decimal 10 with which the idea of that number of anything will be naturally associated by the present generation, until the duodecimal digits shall have become second nature by repeated use. The digit Ɛ is similar to an E; thought of as the initial of "eln" its significance is easily kept in mind.

In place of the decimal point is used the duodecimal pair of points. This is in itself a gain. The former is too inconspicuous to be accurate. Continental practice uses the comma in preference, and frequently relies upon a different font of type for the fraction from that used for the integers, to minimize danger of error. When the duodecimal notation is used for financial purposes a more distinctive mark is necessary, to prevent fraud. A wavy vertical line, such as ≀, or any similar mark, would suffice. In reading the following pages it will be important to note the presence or absence of this pair of points, distinguishing the duodecimal from the decimal systems of notation.

16. From these premises would arise the following notation:

NOTE.—The words outside the parentheses are the names of the duodecimal numbers; they are not to be considered as abbreviations, although they are phonetic abbreviations of the ideas which they are to convey and which are written out in full in the parentheses. Thus, the word "twodz" which is derived from "two dozen," is to be used exactly as the word "twenty," which is a corruption of "two tens," is used in the decimal system. The word "doz" is to be used as we now use the word "ten."

1	One	20..	Twodz (two dozen).
2	Two	21..	Twodz-one (two dozen and one).
3	Three	22..	Twodz-two (two dozen and two).
4	Four	29..	Twodz-nine (two dozen and nine).
5	Five	2δ..	Twodz-dek (two dozen and dek).
6	Six	2Ɛ..	Twodz-eln (two dozen and eln).
7	Seven	30..	Threedz (three dozen).
8	Eight	31..	Threedz-one (three dozen and one).
9	Nine	40..	Fourdz (four dozen).
δ	Dek.	50..	Fidze (five dozen).
Ɛ	Eln.	60..	Sidz (six dozen).
10..	Doz.	70..	Sedz (seven dozen).
11..	Doz-one.	80..	Eighdz (eight dozen).
12..	Doz-two.	90..	Nidze (nine dozen).
13..	Dore-three.	δ0..	Dedz (dek dozen).
19..	Doz-nine.	Ɛ0..	Endz (eln dozen).
1δ..	Doz-dek.	100..	One GROSS.
1Ɛ..	Doz-eln.	1000..	One GREG (one great gross).

In fractions:

0..1	=	one dozt (one dozenth).	$\frac{1}{2}$	=	one-half	=	0..6.
0..01	=	one grosst, or one groat.	$\frac{1}{3}$	=	one-third	=	0..6.
0..001	=	one gregt, or one gret.	$\frac{1}{4}$	=	one-quarter	=	0..3.
	=	one divided by one great gross.	$\frac{1}{6}$	=	one-sixth	=	0..2.
			$\frac{1}{8}$	=	one-eighth	=	0..16.

17. To attempt to handle this notation in terms of the familiar decimal system is, of course, cumbrous in the extreme. In handling duodecimal numbers one rule is fundamental and all-essential:

Think in Dozens.[*]

It is an existing fact, depending not at all upon suppositious future education, that the ordinary person can to-day think in dozens more easily than he can think in tens. The task in attaining familiarity with duodecimal numbers does not lie so much in learning the duodecimals as it does in forgetting the decimals. The difficulty is found, not in thinking in dozens, but in also writing and reading

[*]Also think of 3, 4, 6, 8, and 9 as one-quarter, one-third, one-half, two-thirds, and three-quarters of a dozen respectively.

dozens duodecimally, after having for a lifetime performed the much harder task of thinking in dozens while writing and reading them decimally.

Duodecimal Multiplication-Table.

1	2	3	4	5	6	7	8	9	δ	Ɛ	10..
2	4	6	8	δ	10..	12..	14..	16..	18..	1δ..	20..
3	6	9	10..	13..	16..	19..	20..	23..	26..	29..	30..
4	8	10..	14..	18..	20..	24..	28..	30..	34..	38..	40..
5	δ	13..	18..	21..	26..	2Ɛ..	34..	39..	42..	47..	50..
6	10..	16..	20..	26..	30..	36..	40..	46..	50..	56..	60..
7	12..	19..	24..	2Ɛ..	36..	41..	48..	53..	5Ɛ..	65..	70..
8	14..	20..	28..	34..	40..	48..	54..	60..	68..	74..	80..
9	16..	23..	30..	39..	46..	53..	60..	69..	76..	83..	90..
δ	18..	26..	34..	42..	50..	5δ..	68..	76..	84..	92..	δ0..
Ɛ	1δ..	29..	38..	47..	56..	65..	74..	83..	92..	δ1..	Ɛ0..
10..	20..	30..	40..	50..	60..	70..	80..	90..	δ0..	Ɛ0..	100..

Duodecimal Weights and Measures.

18. To properly develop and advocate the duodecimal system of numbers would fill volumes. Enough has been said to furnish a notation for an illustrative duodecimal system of weights and measures and to show:

(1) How they would harmonize with each other;

(2) How they might be made to harmonize with existing units, tools, standards, habits, etc.

NOTE.—In the tables which follow, the names chosen for the new units are for illustrative purposes only. Probably much better substitutes could be devised.

Linear Measure.

First and foremost, the *foot* and the *inch* could be retained. Their duodecimal expression would substitute for the inaccurate marks (′) and (″), or for the more accurate but more cumbrous substitutes, *ft.* and *in.*, simple integers for feet and a duodecimal fraction of a single digit for the inches, thus:

Decimal.	Duodecimal.
1′ 3″	1..3 ft.
2 ft. $7\frac{1}{2}$ in.	2..76 ft.
5 ft. $3\frac{3}{4}$ in.	5..39 ft.

This could and would be done wherever feet and inches were more convenient than other units, quite parallel to and consistently with the adoption of the following suggestion; which is offered because the development of a complete duodecimal system from the foot as a basis does not result in all that could be desired.

19. The standard unit of length for all English-speaking peoples is the *yard.* Let it be retained as the base for the new duodecimal system of weights and measures quite as the metre is the base for the metric system.

The standard table of lengths would then become:

1 mile	=	1000.. yards	(= 1728 yards = 5184 feet).
1 yard	=	10.. trinches	(= one dozen 3-inch lengths).
1 trinch	=	10.. quarters	(= one dozen quarter-inch lengths).
1 quarter	=	10.. groats	(one groat = one forty-eighth of an inch).

All of these units of length are familiar ones. They are all exact equivalents of present units except the new mile, which is 1.8 per cent. shorter than the present statute mile. But the statute mile is only one of half a dozen different ones, if all civilized countries be included. Thus the present nautical mile varies from 6,080 to 6,088 feet. Taking the average, the new system would stand:

1 nautical mile = 1,210.. yards.

Of the other units, the yard, the foot, and the inch would be used as at present, but with greater facility. The *trinch* (3 inches) would probably be little used as a unit of length; it fits popular needs as little as does the unit decimetre. The quarter, or quarter-inch, would probably become the standard unit for all shop-measurements. Very few machine dimensions would run so large as to make its numbers cumbrous, as is the case when the millimetre is used; very seldom, on the other hand, would any need arise, on the larger work, for a division of it into fractions. When such need di[]d arise, on the smaller work, the standard of shop fractions: $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ inch, etc., for which every workman carries his scale, could be used with perfect ease, as shown by the table below and by Figs. 52 and 53, by means of which he would make his translations by eye instead of by mental or written arithmetic.

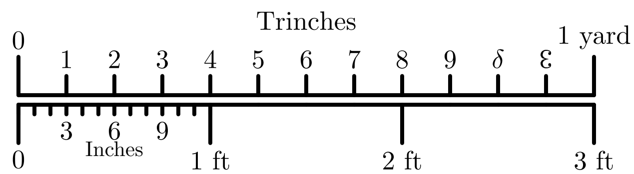


Fig. 52.

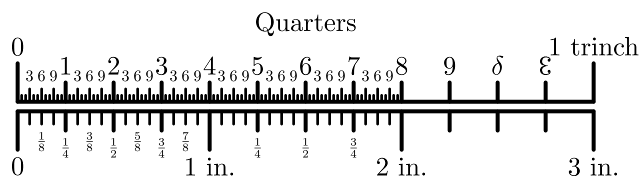


Fig. 58.—The Standard 3-Inch Steel Shop-Scale under the New System.

The *groat* would take the place of the millimetre and the hundredth of an inch. It is closely equal to half a millimetre (0.52916 mm.).

20. In the machine-shop transition to the new system could be made without the slightest change or expense for new tools, etc., except for a new 3-inch steel scale graduated like Fig. 53 for each machinist. New patterns would naturally run on new habits of dimensioning; but old patterns could be produced with no interference whatever with the new. The only obstacle to the adoption of the new system would be the necessity for the learning of the duodecimal multiplication-table by each machinist, which could be done in three weeks of evenings.

The great bulk of machine-work relies upon units no smaller than $\frac{1}{16}$ inch. All such dimensions are expressible in quarters by a single duodecimal place. The same number of digits will express one-third of one-sixteenth, or one forty-eighth of an inch; which dimension would probably be used, in all future work, in place of the sixty-fourth, or one-fourth of a sixteenth, of an inch. Thirty-seconds and sixty-fourths require two duodecimal places. The same number of figures will express divisions to $\frac{1}{576}$, or one-ninth of one sixty-fourth of an inch.

On the other hand, two digits before the duodecimal points suffice to express any dimension short of a yard. The standard divisions on the scales now in use in the shop would be expressed as follows:

$\frac{1}{8}$	*inch*	=	0..6	*quarter.*	$\frac{7}{8}$	*inch*	=	8..6	*quarter.*
$\frac{1}{16}$	"	=	0..3	"	$\frac{5}{8}$	"	=	2..6	"
$\frac{1}{32}$	"	=	0..16	"	$\frac{5}{16}$	"	=	1..3	"
$\frac{1}{64}$	"	=	0..09	"	$\frac{5}{32}$	"	=	0..76	"
$\frac{1}{12}$	"	=	0..4	"	$\frac{5}{64}$	"	=	0..39	"
$\frac{1}{24}$	"	=	0..2	"	$\frac{7}{64}$	"	=	0..53	"
$\frac{1}{48}$	"	=	0..1	"	$\frac{9}{64}$	"	=	0..69	"
$\frac{3}{4}$	"	=	3..	"	$\frac{11}{64}$	"	=	0..83	"
$\frac{3}{8}$	"	=	1..6	"	$\frac{13}{64}$	"	=	0..99	"
$\frac{3}{16}$	"	=	0..9	"	$\frac{15}{64}$	"	=	0..Ɛ3	"
$\frac{3}{32}$	"	=	0..46	"	$\frac{17}{64}$	"	=	1..09	"
$\frac{3}{64}$	"	=	0..23	"	$\frac{19}{64}$	"	=	1..23	"

Until the duodecimal multiplication table is learned these figures seem more confusing than helpful. But even without that preparation, let any shop arithmetician sit down to these duodecimal fractions, *thinking only in dozens*, and trace their relations; he will finish with a strong first impression of the facility and convenience of duodecimals.

The duodecimal statement of these same fractions in terms of an inch is only slightly less clear and facile than the above. Thirty-seconds and sixty-fourths require three duodecimal places instead of two. It is finally to be remembered that these duodecimal expressions for the familiar vulgar fractions can be multiplied, divided, etc., more easily than can decimal fractions—when once the duodecimal multiplication table is learned.

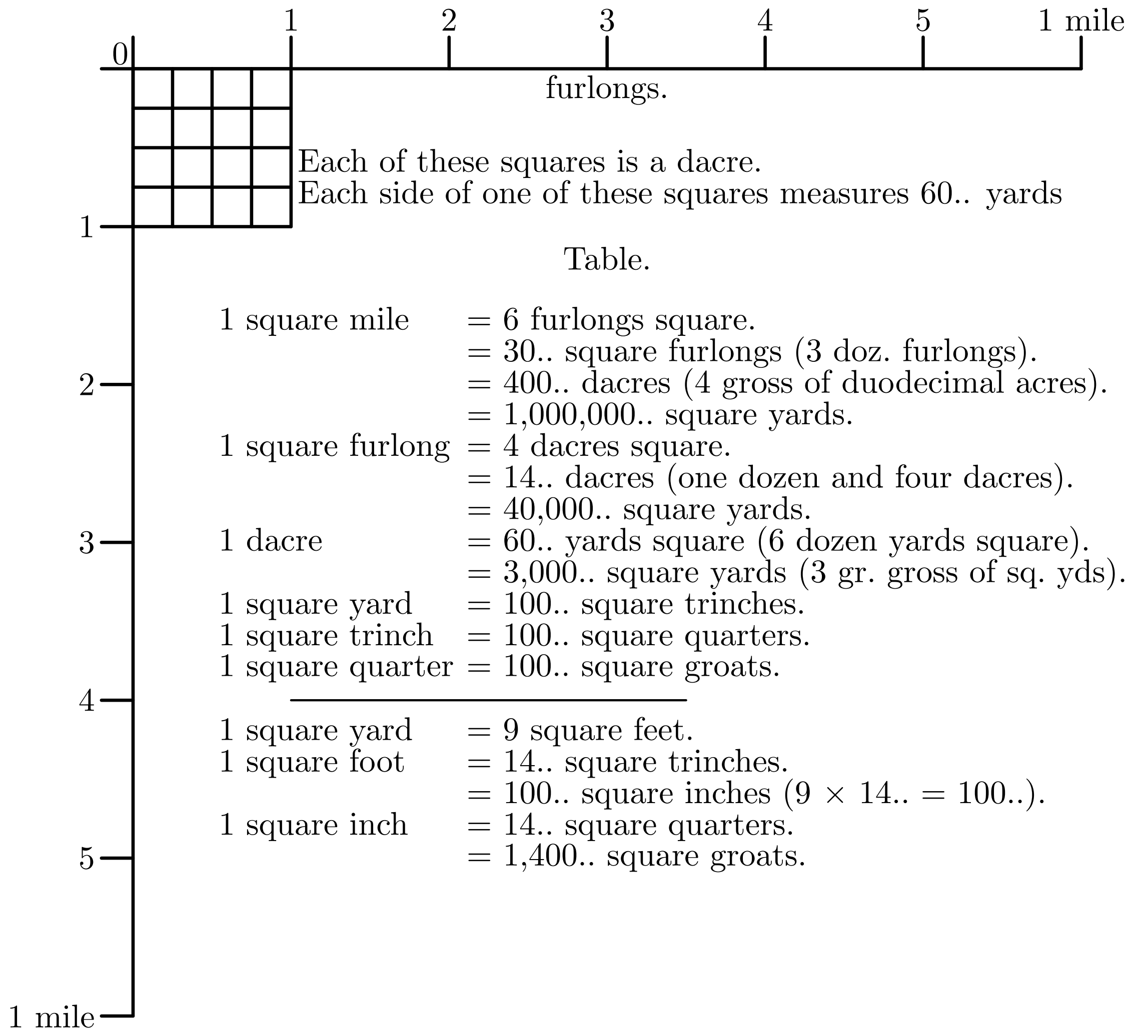
Square Measure.

21. Of all of the tables of measure, square measure presents the most hopeless aspect to the American reformer. The trouble is, not that the system is incapable of reform, but that more finished work lies recorded in it than in any other measure. The great bulk of the territory belonging to this government has been surveyed, divided, and sold by the square mile, section, quarter-section, and acre. To upset this work is a stupendous proposition. It is now more than thirty-six years since Congress adopted the metric system, including its land-measure, yet we hear less today of ares and hectares than we did then. On the other hand, the initiative in any change of measures must originate with the federal government. Should the nation once decide that change were imperative (which is the supposition upon which this paper is based), probably no portion of the task would find itself so unified in control and so quickly accomplished as the alteration of the government land records.

22. A duodecimal system based upon the yard naturally results in a system of square measure something like the following:*

23. All of the above units larger than the yard depart sufficiently from the present units

*To appreciate the figures it must be remembered that, duodecimally, 4 squared = 1..4; that is, one dozen and four. Similarly, the square of 6 is 30.., or 3 dozen (threedz). Editor's Note: This figure has been moved to the following page.



Each of these squares is a dacre.
Each side of one of these squares measures 60.. yards

Table.

1 square mile	= 6 furlongs square.
	= 30.. square furlongs (3 doz. furlongs).
	= 400.. dacres (4 gross of duodecimal acres).
	= 1,000,000.. square yards.
1 square furlong	= 4 dacres square.
	= 14.. dacres (one dozen and four dacres).
	= 40,000.. square yards.
1 dacre	= 60.. yards square (6 dozen yards square).
	= 3,000.. square yards (3 gr. gross of sq. yds).
1 square yard	= 100.. square trinches.
1 square trinch	= 100.. square quarters.
1 square quarter	= 100.. square groats.

1 square yard	= 9 square feet.
1 square foot	= 14.. square trinches.
	= 100.. square inches (9 × 14.. = 100..).
1 square inch	= 14.. square quarters.
	= 1,400.. square groats.

so that interchange would have to be formal and revolutionary. The duodecimal square mile is 3.7 per cent. smaller than the present square statute mile, the proposed linear furlong is 31 per cent., and the square furlong is 71 per cent. greater than at present, and the dacre is 7.1 per cent. greater, or has a side 3.5 per cent. longer, than the acre.*

Volumetric Measure.

24. The proposed duodecimal units of volume are the *standard cubic yard*, the *cubic trinch*, and the *cubic quarter*.

Table.

1 cubic yard	=	1,000..	cubic trinches.
1 cubic trinch	=	1,000..	cubic quarters.
1 cubic quarter	=	1,000..	cubic groats.

1 cubic yard	=	23..	(two dozen and three) cubic feet (23.. = the cube of 3).
1 cubic foot	=	54..	(five dozen and four) cubic trinches (54.. = the cube of 4).
	=	1,000..	cubic inches (23.. × 54.. = 1,000..)

For both dry and liquid measure the proposed duodecimal units are:

(I) The *trink*, or cubic trinch, a cube measuring 3 inches on an edge;

(II) The duodecimal *gallon* of 180.. (216) cubic inches, a cube measuring 6 inches on an edge. Thus:

Dry Measure.		*Liquid Measure.*		
		1 hogshead	=	8 cubic feet (a cube 2 feet long on each edge).
			=	54.. gallons (54.. = the cube of 4).
			=	368.. trinks (368.. = the cube of 8).
		1 barrel	=	4 cubic feet ($4 = 2 \times 2 \times 1$).
			=	28.. gallons (28.. = $4 \times 4 \times 2$).
			=	194.. trinks (194.. = $8 \times 8 \times 4$).
1 bushel	=	8 gallons	=	1 cubic foot (= 2 gallons cubed).
	=	14.. quarts	=	54.. trinks (= 4 trinks cubed).
1 peck	=	2 gallons	=	$\frac{1}{4}$ cubic foot.
			=	8 quarts.
			=	14.. trinks.
4 quarts	=	1 gallon	=	8 trinks (a cube 6 inches long on each edge).
1 quart	=	1 quarter	=	2 trinks (a rectangular solid $3'' \times 3'' \times 6''$).
1 pint	=	1 trink	=	23.. cubic inches (a 3-inch cube).
			=	1,000.. cubic quarters.
			=	1,000,000.. cubic groats.

25. The proposed hogshead contains 64 (54..) of the proposed gallons, as against 63 present gallons in the present hogshead; but as the proposed gallons are $6\frac{1}{2}$ per cent. smaller than the present United States standard gallon, the proposed new hogshead and barrel

*No idea of the simplicity of the above system can be gotten by a glance at these figures without having learned the duodecimal multiplication tables. Thus, to divide 3,000, one of the numbers of the table, by 14, another of them, were they both decimal numbers, would be cumbrous and would lead to an interminable fraction. But when both are duodecimals the task is one of short divisions, viz.:

14..|3,000..

230..

"Doz-four (one dozen and four, or one and one-third dozen) goes into threedz (three dozen) twice; carry four. Fourdz (four dozen) contains doz-four three times."

are 5 per cent. smaller than at present. The proposed new quart, bushel, and peck are also $6\frac{1}{2}$ per cent. smaller than at present, and the trink is the same proportion smaller than the present pint. One trink = 0.44245 litre.

Measures of Weight.

26. The proposed duodecimal unit of weight is that of one trink, or cubic trinch, of distilled water at the temperature of maximum density. Such a cube would weigh 0.97538 pound, or about $2\frac{1}{2}$ per cent. less than 1 pound avoirdupois. Let it be called a *poid.*

Table.

1 (new) ton	=	the weight of one cubic yard of distilled water.
	=	1,000.. poids (one great gross of poids).
1 poid	=	the weight of 1 3-inch cube of water.
	=	10.. *dozts* (dozenths of a poid, or duodecimal ounces).
1 dozt	=	10.. *parts* (duodecimal substitutes for the drachm).
1 part	=	10.. *gregts* or *grets* (of a pound).
	=	2 duodecimal pcnnyweights, (1 p'wt = 6 grets).
1 gret	=	the weight of a quarter-inch cube of water.
	=	4 (new) grains.
	=	200.. pennyweights.
1 poid	=	1,000.. grets.
1 pint	=	4,000.. new grains.
	=	20.. (24) grains, as at present.

In this table the *dozt* is just 30 per cent. greater than the present avoirdupois ounce, or 21.5 per cent. greater than the Troy ounce; but as ounces are not a standard measure of weight, but are used solely as convenient fractions of a pound, this discrepancy matters little. The *part* is 19 per cent. less than the apothecaries' drachm. The proposed duodecimal pennyweight and the new grain are each just 1.27 per cent. greater than their existing standard counterparts. The proposed new ton is 15.7 per cent. less than the present short ton, or 24.8 per cent. smaller than, or almost exactly three-quarters of, the present long ton. Considering, however, that in addition to these two tons we already have in regular use several sizes of marine-registry tons, the miner's ton, and a few more such odd ones, not to mention the metric tonne, it hardly appears that there is an existing standard from which to depart. The convenience of having the ton the weight of a cubic yard of water far overbalances any objection to change from existing methods.

[In the presentation of the paper before the Society the author here refered to a wall-chart, electrotype of which is appended as Fig. 54, displaying the comparative amounts of work involved in calculating the cubic contents, in the several units of volume and weight, of a rectangular tank 24 feet $11\frac{3}{4}$ inches long by 21 feet $11\frac{3}{16}$ inches wide by 5 feet $2\frac{27}{64}$ inches deep, filled with water. The left-hand portion of the chart exhibits the present method, the right-hand the method by the proposed system. It was not urged that this problem was a typical or common one in engineering work; but its awkward association of large and small units in the dimensions brings out graphically the mental saving to be expected in all computation, which is the only argument in favor of the metric system which has been sufficicntly cogent to insure its adoption, viz., in scientific work.]

Money

27. If our arithmetical notation and our standard weights and measures unite in becoming purely duodecimal in character the

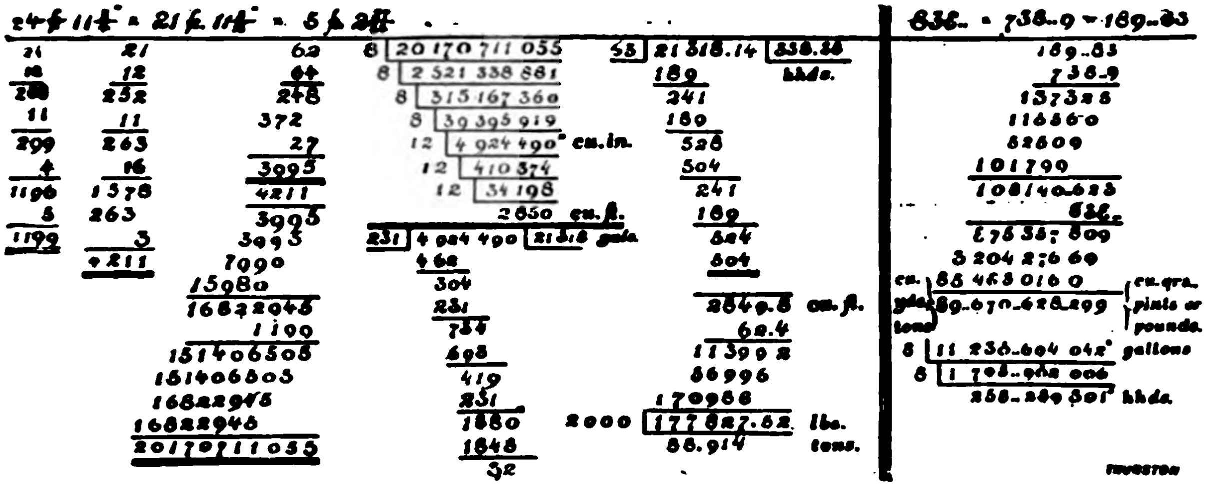


Fig. 54.

monetary system is bound to follow. This proposition is not so revolutionary as would at first sight appear. The standard of value, the dollar, and all of its unit representations would remain unchanged. All bills of five dollars or higher denomination would naturally be called in and their equivalent issued in denominations of three, six, nine, doz, gross dollars, etc. But this process could be as gradual as desired. Under duodecimal notation five and ten-dollar bills would be inconvenient, but they would be useable.

As to coins, the half-dollar and quarter-dollar would remain unchanged. The dime, the nickel, and the cent would. have to be retired. In their place would be issued fractional currency under the following plan:

1 dollar	=	10.. bits	(one dozen bits of $8\frac{1}{2}$ cents value each).
1 bit	=	10.. groats	
1 groat	=	10.. grets	(for purposes where the *mill* is now used).

The probable coins would be:

The silver half-dollar	=	6 bits	=	60..	groats	(= 50 cents);
The silver quarter-dollar	=	3 bits	=	30..	"	(= 25 ")
The silver *bit*	=	1 bit	=	10..	"	(= $8\frac{1}{2}$ ")
The nickel *half-bit*			=	6	"	(= $4\frac{1}{4}$ ")
The copper quarter-bit piece			=	3	"	(= $2\frac{1}{16}$ cents);
The copper *groat*						(= 0.7083 cents.)

28. Aside from its duodecimal advantages, this schedule presents two minor points of advantage over the present decimal one:

(I.) Change for a quarter could ordinarily be had in a single convenient denomination—that is, in three silver bits, whereas now it requires two denominations, dimes and nickels, to make it. (The practical objections to relying upon nickels alone for changing quarters are obvious.)

(II.) The progress of business toward finer margins and lower prices is steadily making the cent too large for many retail transactions. The smaller value of the groat harmonizes with this need.

29. But any complete comparison between the two systems must amount to the fact that we should never perceive any conscious difference in using the new system, although its economy of time and effort would be there nevertheless. The period of novelty due to its introduction would be less than that experienced by an American using British money for the first time.*

30. This completes the list of essential measures to be affected by the fancied revolution. From it will be plain that a line of progress is open before us which will accomplish the following results:

(a) Harmony between the systems of measures and of notation, which is all that the metric system has to offer;

(b) Greater facility in computation, which is what the scientific world especially desires;

(c) Division of units by twos and by threes, which is what the people especially desire, for they cannot live without it;

(d) A large measure of consistency with existing standards, it being absolute and accurate in linear measurements and so closely approximate in weights and measures of volume as not to appreciably disturb popular conceptions;

(e) Possibility of a gradual transition from one system to the other, not without great cost, but without catastrophe.

Thus, as to this last, all English-speaking peoples ought long ago to have united in making the standard gallon contain 216 cubic inches, or a cube measuring 6 inches on an edge. The standard pound ought to be the weight of a pint of distilled water at maximum density. The ton ought to be a cubic yard of the same. These changes can be undertaken to-day, to an advantage well worth any disturbance they might create, whether any duodecimal system of numbers be contemplated or not. And yet, when these things are once done it will appear that the bulk of the cost of the adoption of duodecimal notation, to the industrial world, at any rate, has already been overcome.

Discussion

Mr. John D. Riggs.—Our present system of feet and inches for linear measurement with inches divided into halves, quarters, eighths, sixteenths, thirty-seconds and sixty-fourths is just a little inconsistent. If we can compare a dimension of say a sixteenth of an inch with the inch as our unit, and get a clear conception of its magnitude, then why can we not compare the inch unit with a dimension of fifteen inches and avoid the use of the foot altogether? In practice does not a man come to know the value of a sixteenth of an inch as a unit, and should not this unit have a better name than it now has? When this unit gets the name that is due it the sixty-fourth can be read as a quarter of a sixteenth of an inch, and the millimetre will not seem so small when compared with this old unit with a new name.

*Therein is suggested another palpable opportunity for advance. The American five-dollar piece, the British pound sterling, the German twenty-mark piece, and the French 24-franc value ought to be made equivalents. Then we should have:

American.		British.		German.		French.
5 dollars	=	1 pound	=	20 marks	=	24 francs.
1 dollar	=	4 shillings	=	4 marks	=	4.80 f.
3 bits	=	1 shilling	=	1 mark	=	1.20 f.
1 bits	=	4 pence	=	....	=	40 c.
3 groats	=	1 penny	=	....	=	10 c.

The proposed system based on the number 12 seems to be very fortunate, in that it brings in the factor 3 just often enough to suggest a new unit-name, and thus avoid such fractions as thirty-seconds and sixty-fourths. But as the substitution 12 for 10 eliminates the objectionable feature of the metric system, why not base this system on the standard metre instead of the yard? Dividing the metre into 12 parts we get a unit about equal to the width of a school-boy's hand, and for the present we may call it a (metric) hand, dividing this again by 12 we get a dimension nearly equal to the diameter of the ordinary round lead-pencil, and which we may call a pencil. Dimensions smaller than this might be expressed as fractions.

Some will ask, why make a change to the metre and not get the metric system after all? But if in making this change we can anticipate the next one and thus make the two changes as one and avoid most of the confusion, we will have gained a point. If the second change should never be made by the other nations, we will still be based on the same standard, and our subdivisions will have a very simple relation to theirs.

After all, the thing we are after is, in my opinion, to be able to comprehend dimensions and measurements. That system is best which will enable designers and workmen to comprehend what stated dimensions represent.

Mr. George W. Colles.—This paper is something more than an admirable summing-up of the present status of the Weights and Measures question, and a step forward. It is a step forward in the right direction, and I can say with truth, that, of the scores and dozens of schemes for new weights and measures systems which have been proposed, and many, if not most, of which I have seen, this is the very first of which that can be said. I do not say it is the first "rational solution" of the problem, but is certainly the most rational solution that has yet been proposed, and therefore well deserves its title.

Some years ago, I had the honor to present to this Society a paper which, though bulky, was yet incomplete, for while part of the paper was devoted to the history of weights and measures, the remainder was devoted to the objections to the metric system, and having finished this part of the work, I found it necessary, on account of the magnitude of the work, to postpone a special consideration of the duodecimal system of weights and measures until another time. Nevertheless, as was pointed out in that paper, and as the title of the paper itself implies, the special consideration of the duodecimal system itself and its possibilities was merely postponed. The sequel to that paper which I then had in my mind and which I have had in my mind ever since, was to outline a scheme of improvement on precisely the same lines as that proposed by Professor Reeve. As I have never found an opportunity to undertake this myself, it is with very great pleasure that I see that it has been undertaken by some one else and at a critical juncture, because it is evident that, in order to stem the tide which has apparently set in favor of the metric system in many circles, it is necessary to give a serious consideration to our own present system, which is evidently capable of great amelioration.

As Professor Reeve has taken up the task and presented a rational scheme of improvement, I believe it will not be without interest to outline in a general way my own ideas on this subject, which were obtained largely during a consideration of the historical matter on the subject of weights and measures and which enabled me to arrive, though by a somewhat different path, at almost identical conclusions with his own. My investigation of past history shows that it is not the case that our present congeries of independent measuring units are in fact independent of one another, and merely selected at random without reference to their mutual relation. The fact that Professor Reeve has been enabled to work out so admirable and well-fitting a system from our present units has its *raison d'étre* mainly in the fact that he has unconsciously returned to the original relations which these measures bore to each other. The mass of evidence on this point is very great, and could it all be presented together, would hardly fail to be convincing. While I speak at present wholly from memory, I think I can safely lay down the following as among the mutual relations of our weights and measures:

1. The gallon was 216 cubic inches, or one-eighth of a cubic foot. Our present gallon of 231 cubic inches is an anomaly, like all the rest, brought about by ill-advised legislation on a false basis.

2. The pint was a cube of three inches on an edge, or what Professor Reeve calls a "trink." It naturally follows that 8 pounds make a gallon, and 64 pounds or pints a cubic foot.

3. The bushel was 2,160 cubic inches (U. S. standard bushel = 2,150.48 cubic inches), that is to say, was 10 gallons, or $1\frac{1}{4}$ cubic feet. The *raison d'étre* of the bushel is that it is an equivalent in weight of wheat to the gallon, that is to say, a bushel of *wheat* weighs approximately the same as a cubic foot of *water*, or 64 pounds. Not exactly, perhaps; but the approximate ratio of 4 : 5 between the specific gravity of wheat and water (or rather wheat and wine, the two chief articles of commerce) was so convenient for ordinary measurements, that it was adopted here as in a number of other cases, some of which were referred to in my paper before mentioned.

4. It should be remarked that there was at some time a special measure of one cubic foot for liquids, though what it was called at various periods is uncertain. It was called *amphora* by the Romans and was the universal measure of capacity in bulk, as, for instance, in measuring the displacement of ships.

5. The *ton* (formerly the same as *tun*) was formed by doubling and redoubling upwards from the gallon, forming the intermediate measures of the barrel and hogshead, and the ton, therefore, was 32 cubic feet (*not* one cubic yard), or $2'' = 2{,}048$ pounds (or pints). The figure 2,000 is a corruption assumed for convenience in calculation by the decimal system, but it spoils the harmony of the original system.

6. The *mile* as a lineal measure is an anomaly, and not a part of the original system, being, as its name denotes, "mille passus," i.e., one thousand double paces of five feet each, therefore partly founded on an independent base (the natural pace) and partly on the decimal system.

7. Neither does it appear that the *yard* is a part of the original system, but this was a Teutonic measure which was grafted on subsequently. Therefore, so far as concerns Professor Reeve's coincidences in duodecimals between the yard, mile, and quarter inch, they are purely accidental. The foot was the actual standard of the ancient system as it is in all civilized countries to-day, while the yard or its equivalent is limited to a few, and has but comparatively limited application. The foot is still used to the exclusion of the yard in the great majority of cases, and has been from the first divided into 12 inches, 144 lines and 1,728 points—therefore strictly on the duodecimal system.

8. The above consldcrations are sufficient at least to show that the units of the ancient metrological system were strictly coordinate one with another, although the subdivisions and multiples of these units were not strictly duodecimal, but on the contrary partly duodecimal and partly octonary. Most of these points are referred to incidentally in my paper before mentioned.

While I have no thought of the desirability of returning to a system merely because it is ancient, still it is my firm belief that it will prove far easier to return to the original system than to undertake a new departure to exhibit relations between units which are entirely foreign to it and merely accidental.

It appears also from this standpoint, that while Professor Reeve is strictly correct in saying that we should long ago have adopted a standard gallon containing 216 cubic inches, he falls into error in making a *bushel* equal to one cubic foot, or 8 gallons, instead of $1\frac{1}{4}$ cubic feet, or 10 gallons, which it approximately is, and which would amount to an extremely small and comparatively unnoticeable departure from the present bushel, and consequently also in its subdivisions. Professor Reeve is, of course, welcome to retain the cubic foot as a dry measure, but he must not call it a bushel. Similarly the proposal to call a measure of 1,728 pounds a *ton* not only does unjustifiable violence to the proper relations (with respect to which a 2,000-pound ton is much more proper), but he also departs far too widely from our present ton to avoid an intolerable confusion. There is no ton now of less than 2,000 pounds, and while a measure equal to three-quarters of the present long ton or metric ton may be convenient, it must not be called a ton.

Perhaps I may be permitted to add to the already very excellent setting-forth of the matter in Professor Reeve's paper a few general considerations on the question of altering weights and measures.

1. The first question to be considered, when a proposal for metric reform is made, is, shall we sweep away altogether the old units and replace by new ones, or shall we amend and improve the old system? And in so amending, is it better to retain only the basic units for the different quantities, or shall we make small and insignificant changes as far as possible in the special subdivisions, so that they shall accord with one another on the system we propose? Experience shows the difficulty, nay almost impossibility and worse than uselessness of attempting the first course. The very first principle to be laid down is to adopt the very fewest new units possible, and the second is that, where they are adopted, they must be commensurate with the old. As to making small changes in secondary units and calling the changed units by the same names, great objections have been offered owing to the confusion necessarily engendered as to exactly what is meant by a name, yet, on the whole, I think this is far less an evil than the introduction of an absolutely new and discordant system, and far less dangerous than the introduction even of new units which accord more or less with the existing ones.

2. Not less a point for consideration is that the proposed reform must be capable of being adopted gradually, little by little and piece by piece, and not by any sudden and revolutionary change, of which the metric system is a perfect example, and which has the result of merely introducing discord, which it never replaces or drives out. Now Professor Reeve's plan is just such a system as, contrary to the metric system, may be adopted little by little and with the least possible violence to popular uses and customs, though undoubtedly requiring the aid of a certain amount of legislation. It is not by any means necessary that it should *all* be adopted to secure the improvement of the present system, but the adoption of any part by itself will improve the system, leaving the question of the adoption of a further part optional at any time in the future. Nor is it necessary that all the proposed units be adopted precisely as outlined by Professor Reeve; but this should be the subject of consideration by a commission of highly-skilled metrologists of the principal English-speaking nations before anything is done, if that be possible; although I do not mean to say I would disfavor a single well-considered step by the United States Government alone, as international commissions are so seldom fruitful of results.

3. As to the proposed duodecimal notation, I must admit that is a question I have never seriously considered. Such a system has been proposed before by many mathematicians and even actually used. That it is actually easier when once learned is beyond a doubt, and yet it is equally true and more important to note that the decimal system is so deeply and universally rooted in the mind of man, that it would be nearly impossible to eradicate. I feel that, while scientists may use this to advantage if they do not come into contact with the decimal system, yet the lat-

ter would introduce such confusion in their thoughts, that they would find themselves perforce compelled to abandon the former. It seems to me, in fact, that even Professor Reeve has underrated the difficulties of making a change, as history proves that people hold on to their units with a firmness that nothing can shake, albeit such firmness is nothing after all but a mere dead resistance of a magnitude practically insuperable by the legislator.

4. As to money, our present unit has, of course, no actual relation whatever to any metrological system, old or new. Professor Reeve's division of the dollar is therefore purely arbitrary and in so far objectionable; although that it would be more convenient than the present, goes, of course, without saying. A good instance, however, of the point last referred to, as to the difficulty of changing units, is that suggested by him in the *appromimate* equivalents of the American half-eagle, the British pound, the German 20-mark piece, and the French 24-franc value, which, of course, by all common sense ideas, *ought* to have been unified long ago, but, as a matter of fact, this has been tried and given up as a hopeless task, as no agreement between the different nations concerned could be reached. The British nation, for instance, would undoubtedly be very glad to have the United States, Germany and France, change their units to correspond with the pound sterling, but they themselves would not be willing to change the value of the pound by the twentieth part of one poor scruple, as has been shown by the agitation for decimal currency and on other occasions in Great Britain. As well might it be tried to agree upon a common language.

5. One of the greatest objections to the system proposed, not only of duodecimal notation but of duodecimal weights and measures, is the introduction of new words. The experience with the metric system showed what an insuperable prejudice the popular mind has to such innovations. This must be counted among the apparently unavoidable accompaniments of any important change in weights and measures.

In conclusion let me say that I do not think this question should be treated lightly or apathetically. There is no valid reason why the Committee on Coinage, Weights and Measures of Congress should continue to grind out, year after year, the same old bulletins and the same weather-worn arguments in favor of the metric system, and bills to make it compulsory. Could sufficient interest be aroused on the *other* side of the question, and this Committee be got to even consider the amendment of our present system in a rational manner, there is at least no doubt but that a much greater advantage would accrue to the public. The fact that hundreds of men, clubs, societies and other bodies can be got to endorse the metric system in a general way, or to cite points in its favor, as in the recent symposium called for by the Franklin Institute, seems at first disheartening to those of us who believe we see its defects; yet they are in fact of little more importance than the popular endorsement of a patent medicine, because very few of such persons as have endorsed it, however able in their special department in life, have ever given the question of weights and measures and notation a serious and prolonged consideration. The fact that they cite the decimal divisions as the great advantage of the metric system, whereas in fact, they are the supreme objection to it, shows fairly well that this is the case. I only wish that more of our practical scientists could be got to try the duodecimal system, especially with its accompanying notation, as Professor Reeve has done.

Mr. H. H. Suplee.—In the first place I wish to congratulate Professor Reeve on the good work that he has done. I think the applause which greeted him showed that many of the audience agreed in some of his points, at least. The only remark I wish to make now is to call attention to the fact that a somewhat similar system was prepared a number of years ago by the veteran John W. Nystrom, only that he based his upon 16 instead of upon 12. The system was worked out at short length in his well-known "Engineers's Pocket Book," although I believe it has been left out of the recent editions, and I think he prepared a complete arithmetic on that system and also used it in his treatise on "Steam Engineering." I think his work in that direction was brought to a close by his death rather from any change of opinion on his part. He continued to be an advocate of it to the end, and I think that Professor Reeve has taken up that branch of the work in an excellent manner, and I hope will carry it through.

So far as the workman in the shop is concerned, it does not matter very much what system he uses, since he must work mainly to gauges anyhow. The dimension for him is, and should be, merely the name for the gauge, whether it is in the decimal or duodecimal system is a matter of minor importance.

Professor Reeve.—I should like to know a little more definitely than I have yet discovered what is the verdict of the Society upon this proposition. To make it of any value to the profession it must be raised from the level of a suggestion, where it now stands, to a condition where it can be tried, upon a limited scale at least. That means a large amount of decidedly tedious labor. I have had no time to undertake that. I have had no basis. I do not feel that I now have any basis for doing it. If there is no general opinion upon the part of the profession that progress in this line is valuable as well as possible, it is hardly worth either my while or that of any one else to prepare those tabulations of a numerical sort which are essential to the first trial of the plan. I will not say that I shall not some day undertake the task, but I certainly shall not do it immediately, and I should feel very little like looking forward to it if there is no general expression of approval. I would ask, rather as a personal favor, that there be either approval or disapproval in so far as there can be.

Mr. Wilfred Lewis.—I would like to ask Professor Reeve whether he could not give us a comparative statement of the relative merits of this system which he proposes on the system referred to by Mr. Suplee, in which 16 was taken as a base instead of 12—whether there are not advantages in favor of 16 which do not apply to the duodecimal system?

Professor Reeve.—The reply is simply that the history of the world, as Herbert Spencer puts it, has shown, by the survival of the fittest, that when a man wishes to divide a thing he first divides it by 2. If the division by 2 results in too large a quantity, he next divides by 3. If the division by 3 results in too large a quantity, he divides by 4. By that time the point where simple, easy division is carried on by the eye or by estimate has been surpassed. Beyond that it does not make much difference whether divisions run by 5, 6 or 7, or what they are; but to leave out the factor 3 would cut us off from two things: in the first place a very valuable division, smaller than a half and larger than a quarter, and which appears very prominently in this multiplication-table when you come to analyze it. That is the 3d; or 4 units on the basis of 2 parts. Secondly, we have got to adhere to present standard units of length. I think that nearly all of us are agreed on that, and the present standard of length is the foot and the inch. The factor 3 enters in everywhere until we subdivide the inch; then only do we adhere to the binary division. At any rate, the foot and the inch and the yard are inseparably connected with the factor 3.

Mr. F. A. Halsey.—I would like to ask Professor Reeve regarding the feasibility of using the two systems conjointly through a long period of time, for therein, it seems to me, is the fundamental difficulty. I do not suppose there is any one who has given this subject any serious attention who is not convinced of the advantages of the duodecimal system. I suppose the actual, tangible advantages of that system compared with the imaginary advantages of the metric system would stand in the ratio of possibly 100 to 1, certainly 10 to 1. I think that Professor Reeve makes the same mistake as the metric advocates in assuming the chief difficulty to lie in learning to think in the new system. It seems to me that the chief difficulty lies in the fact that our system of notation, like our system of weights and measures, is "tied irrevocably to the past." What I mean is that our numerical records of all kinds, regardless of nationality, geography, language or age, are all based upon the number 10, and it seems to me that to introduce this change would introduce confusion that would last for a thousand years. That would be the case, unless the two systems could be used conjointly.

Professor Reeve.—In reply to that I would say that I anticipate that at the start, certainly, and for a long time probably, they would be used conjointly. The place where they would be used first would be the drafting-room. Draftsmen are slaves anyway, and they would have to adopt the system if they were told to do so. If the drawings had to be labeled in duodecimal units, then the draftsmen would soon find it most convenient to compute in duodecimals; but I can easily imagine a drafting room in which the men are not required to do that; in other words, where they would use decimals for the attainment of these duodecimals, until they found it easier to do the opposite. For instance, they would say: "Seven times 8 is 56, and 56 is 4 dozen and 8;" they would then put down the 4 and the 8. They would continue to do that until they got tired of doing it, finding it easier, as I very promptly did, to think in dozens and to say: "Seven times 8 is 4 dozen and 8," as mechanically as one now says, "7 times 8 are 56."

In the shop the transition would be much more gradual. The machinist needs to know very little about any change in units. He uses exactly the same units, the same gauges, he uses the same dimensions in everything. He uses this lower side of his rule (pointing to the lower scale of Fig. 53) which he now uses, just as long as he finds it easier than to use the upper scale. When the shop-drawings come in with a dimension stated in the new units, he picks it out by reading 3, 6, 9, etc., on the new scale, except that the new scale is simpler. At the end of this process he finds that he has arrived at one of his old-fashioned, familiar dimensions. When he finds it easier to work to 3, 6, 9 directly, without translation into the old scale, he will do it; but he can do either. Any man in the shop, as I imagine it, can take his choice between the duodecimal way of handling the old measures, or the old way of handling the old measures, whichever way is the easier. I do not anticipate the new unit becoming in any way a fixed standard. Men would probably slowly acquire the habit of thinking in quarter inches instead of thinking in inches, but in the meantime the length would be the same and the tool would be the same. The 3-inch length I do not anticipate becoming active in shop-measurements, except in one way: Tapers are always stated as so much to the foot. In the new combined scale those ratios may appear and be used either as inches to the foot, as in the old-fashioned scale, or as trinches to the yard, or as quarter inches to the trinch. The taper may be marked off and set in sixteenths or in these new marks, or in any other way. The lengths are the same, the proportions are the same.

The transition to better methods by any duplication of systems will undoubtedly bring in confusion and error; but duplication is absolutely unavoidable if any progress is to be made. It seems to me that the confusion and error in the method proposed would be exceedingly small. In other words, the price paid would be exceedingly small, as compared with any other possible outlook away from the present system.

Mr. Halsey.—I asked the question in the sense of numerical calculations and records rather than in the sense of measurements. Imagine a bookkeeper to have made the change in his books. How much confusion would result from his references to the old books in the old system, from his constant receipt of bills, price lists, etc., from those who had not made the change, and from the necessity of his making out bills in the old system for those who could not read them if made out in the new. It seems to me that for a long period of time we must all have an equal facility in the use of both systems, and that, unless this is possible, the change is impossible.

Mr. Reeve.—In the dollars it would make no difference whatever. In the cents it would. He would have to translate the cents.

Mr. Halsey.—Do you rely upon the double decimal point to distinguish in which system a sum of money is expressed?

Professor Reeve.—Not in monetary transactions; I would not. A man could then easily raise his check by simply putting on a double decimal point. But it is easy to substitute a mark in monetary transactions which could not so easily be changed. But where the two duodecimal points were relied upon I see no probability of greater error therefrom than now occurs from reliance upon a single decimal point.

As for other computations, the man in the drafting-room chooses either side of the chart (Fig. 54). He can compute in the old measures and simply translate his final result into the new one, which is a compromise process; or he can accept the new system and calculate by the method shown on the right-hand side of the sheet. Of course, while he is taking his choice and using both systems at once, there will be a number of mistakes. I might say, however, that in preparing that chart, which was prepared rather hurriedly, the first computation developed three mistakes; but they were all on the old system, on the left-hand side of the line. While carrying out the duodecimal multiplication at the same time that I was handling the decimal numbers, as you see, there was no mistake in the duodecimal multiplication. Within the first week that you try half a dozen times, half an hour at a time, to multiply and divide duodecimals, you will realize that it is very much easier to think in dozens than in tens. It is easier and more accurate. You will have to take my word for that.

Mr. McGill.—I would like to ask Professor Reeve how he would change that scale (Fig. 53) into thousandths? There are lots of us who do not use 64ths, not once in a week, as a rule.

Professor Reeve.—The thousandth seems to be a unit by itself. Whenever the machinist works to a thousandth he does not stop at a thousandth. It is not accurate enough for that kind of work, and when he needs a fraction of a thousandth that fraction is not a ten-thousandth. He does not work to so many thousandths and then three ten-thousandths over, for instance. He works to so many thousandths, half-thousandths, or quarter-thousandths. The words "a thousandth" is a unit. So long as that unit is used I do not think that there would be any particular benefit in trying to translate it into the new system. If the drawings were stated in that way and the gauges were made in that way, they would be used in that way. In screw-threads there would be no change whatever. They are stated so many per inch. The inch we can handle as well in this new system as we do now. The new substitute for the thousandth of an inch, as new drawings come in, would be found in the second duodecimal place, considering the quarter-inch as the standard unit. The second duodecimal place beyond the quarter-inch is $\frac{1}{144}$; of a quarter-inch, or $\frac{1}{576}$ of an inch, or one-ninth of a sixty-fourth (see Fig. 53). Now the 576th of an inch is accurate enough for nearly all fine work—not so fine as to need fractions of a thousandth—and you get that degree of accuracy with no more figures than are needed to express either thirty-seconds or sixteenths. But if greater accuracy be needed, the use of another, or third, duodecimal place permits the expression of dimensions as fine as one-twelfth of the 576th just mentioned, or about seven times as fine as the thousandth of an inch; and I think it will be admitted that very little machine-shop work goes any finer than that. Moreover, to remind one of how frequently the advantages of duodecimal notation crop out, if a man had been working to the second duodecimal place from the quarter-inch, either in shop or drafting-room, and finds that he needs greater fineness, he is not compelled to either limit himself to halves or fifths, as he is in using thousandths, or else add a vulgar fraction on the end of a long decimal fraction; instead, the third duodecimal place permits him to work to halves, thirds, quarters or sixths of his last smallest unit without incurring vulgar fractions. So that in a shop where thousandths were used the new system would offer a more convenient parallel which would soon drive the other out. But in so far and so long as thousandths were used I should not contemplate any attempt at handling them on the new system. They simply would gradually die out of use.

Mr. Colles'* suggested idea that the coincidences with existing units of weight and volume which developed from the foundation of a duodecimal system upon the existing units of length was something more than a coincidence, had already impressed itself upon the author during his investigation of the question, and had been orally discussed with some of his friends. But the argument to be drawn therefrom did not seem to be sufficiently defined or cogent to warrant its inclusion in the paper. It is nevertheless of great interest, and Mr. Colles' able presentation of it from the historical standpoint is valuable.

As to the author's suggestions regarding a new bushel, or similar new modifications of old units, as of the new names suggested for the numerals, they were included merely as illustrations, to render the proposition concrete. In approaching this entire subject one cannot avoid being impressed with the utter futility of attempting to accomplish any real progress by proving by argument that any particular system is so good that every one ought to adopt it. The only proposition which can attain universal adoption is one so simple and concrete, carrying such patent advantages, that each individual who meets it may adopt it with profit, without waiting for others to realize its advantages. Such a proposition is that for dividing the 3-inch scale duodecimally and using corresponding duodecimal arithmetic in the drafting room—or such a proposition it would be were the necessary accessories in shape to be laid before the Society or the public. They could be produced at much less cost of effort and money than has already been expended upon many similar projects which failed. But until they are produced the topic must remain in the form of a suggestion only. But in such a suggestion it is not only proper, it is necessary, to point out that the adoption of the first few steps, for the sake of their immediate convenience, would not land the pioneer at a dead-end, out of touch with other systems and unable to keep near his fellows without retracing his steps, but would open before him additional opportunities for convenient modification of existing units into consonance with what he had already done, when he felt that they, too, offered advantages fit to warrant the change.

Thus, as to the new ton, if it be supposed that duodecimal notation has been adopted within a certain community (which might consist of a single shop or circle of shops, such as this country now has several of), the alternative lies before it of either calling the present short ton 1,138.. (= 2,000) lbs., and the present long ton 1,368.. (= 2,240) lbs., or of making use of the duodecimal 1,000.. lbs. as a new unit. So long as outsiders using the old system are in the majority, it will pay to do the first, translating from decimal to duodecimal numbers by the translation-tables which must be relied upon so long as both systems are in use. Finally, however, it must prove to be more convenient to use the 1,000..-lb unit, and for it then will be found a name.

*Author's closure, under the Rules.

Whether this unit be smaller or larger than what would seem to be an ideal size for a ton will have nothing to do with the final result.

As to the author's suggestion as to what abbreviations of the duodecimal, or "dozenal," numbers might be the result of long use, he would say that he has already found reason to regret having made it. The non-technical, and to a certain extent the technical, press has seized upon these strange names as constituting the core of the idea. The author, in what use he has made of duodecimal numbers, has never found reason to depart from the simple names of "four dozen and eight," etc. They carry an already familiar idea in an only slightly strange manner, and are very readily adopted and understood.

This document was digitally reset, with all new figures (except Fig. 54), by Donald P. Goodman III for the Dozenal Society of America (http://www.dozenal.org) in March 11Ɛ↊ (2014.). Fig. 54 was simply copied from the original text. On page 7, the word *did* was misprinted *dild*; we have correct this, but marked the deletion with "[]". We have also eliminated a footnote in the beginning listing some other works from the TRANSACTIONS. It is hoped that this work will be helpful to those learning about dozens and studying the history of the dozenal idea, which appears to go back quite a bit farther than many believe.